Lecture 10: Assembly

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Learning objectives

1. *Problem statement*
2. *Introduction to Design For Assembly (DFA) / Design for Manufacturing (DFM) principles*
3. *Cost of an assembly*
4. *Illustrations*
5. *Extra:* mechanism theory – duality mobility / degree of hyperstaticity

Chaplin: 'Modern times'

Assembly: problem statement

- The majority of products are made of **individual parts assembled together**...
- Often, not an *ideal,* but a *necessary* solution...
  - **Combine** materials with different properties
  - **Mobility** in the structure: mechanism
  - **Replaceable** parts
  - *Etc.*

Assembly and functions...

- Just to **hold** to part together...
- To **precisely position** an element versus another one
- To **link** two parts together (that can move relatively to another one)
- **Assembly implies additional manufacturing cost** // may or may not be balanced by the gain in complexity for an assembled part.

DFM vs DFA

Design for Assembly (DFA)

- concerned only with **reducing product assembly cost**
  - minimizes number of assembly operations
  - individual parts tend to be more complex in design

Design for Manufacturing (DFM)

- concerned with **reducing overall part production cost**
  - minimizes complexity of manufacturing operations
  - uses common datum features and primary axes

Similarities between **DFM & DFA**

- Both **reduce** material **overhead** and **labor** cost.
- Both **shorten** the product development **cycles time**
- Both **seek standards** to reduce cost.

Note that DFM & DFA are sometimes referred under a single name DFMA.

Rationale for these methods

- **Quantitative** method to assess design
- **Communication tool** between engineering disciplines: design, manufacturing, production.
- **Involvement of manufacturing in the early design phases.**
- As **75%** of the product cost is determined during the 'engineering/design phase', hidden waste areas are identified **earlier** before committing to a design.

Design for assembly **(DFA)**

'Design for assembly (DFA) is a process by which products are designed with ease of assembly in mind'

- A variety of **design rules** to:
  - Lower costs by reducing number of parts
  - Optimizing parts for simpler assembly
  - Reducing process costs
  - Optimizing manufacturing sequence
- An ensemble of **trade-off**
  - **Fewer parts** vs **performance** / robustness
  - **Process** vs **manufacturability**

DFMA: principles

Importance

1. **Minimize part count**
2. Design parts with **self-locating features**
3. Design parts with **self-fastening features**
4. **Minimize reorientation** of parts during assembly
5. Design parts for **retrieval, handling & insertion**
6. **Emphasize** 'Top-Down' assemblies
7. **Standardize** parts...
8. Minimum use of fasteners
9. Encourage modular design
10. Design for component **symmetry** for insertion

Boothroyd-Dewhurst method

This method starts from an **existing design** which is iteratively evaluated and improved.

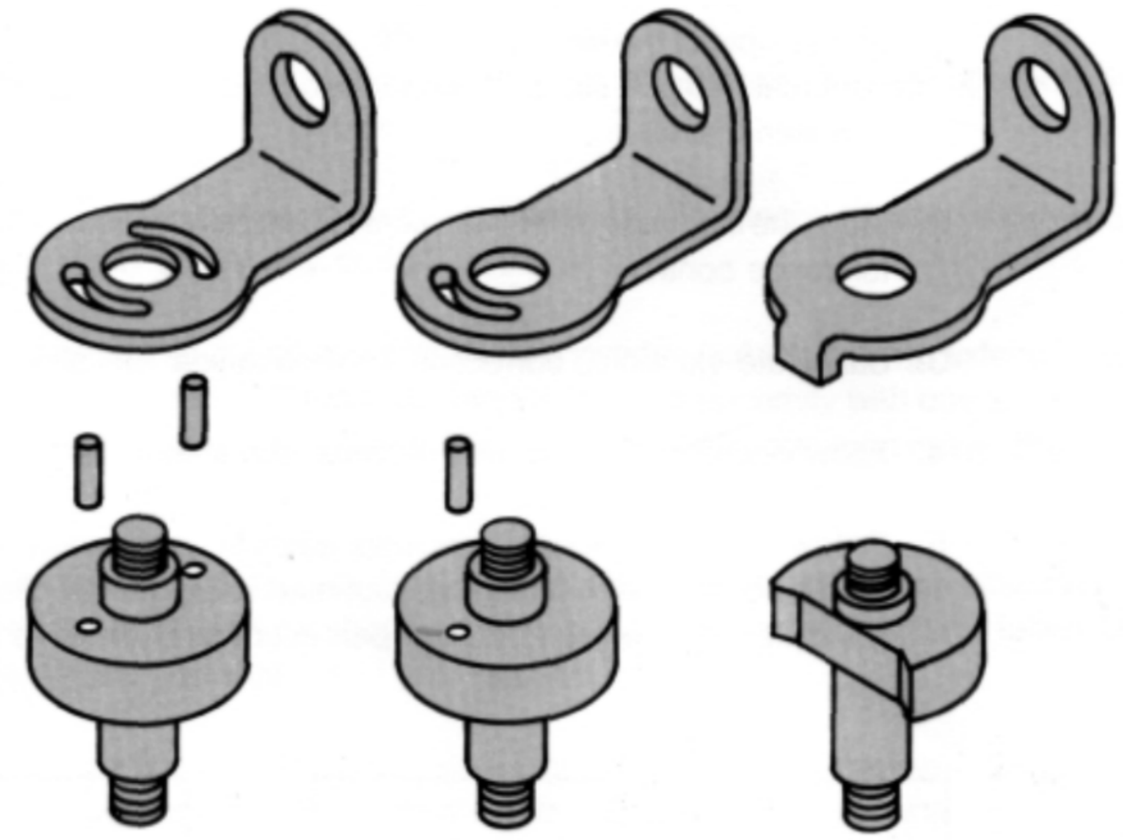
Based on two principles:

- – *application of criteria to each part to determine if it should be separate from all other parts.* ***(functional evaluation)***
- – ***estimation of handling and assembly costs*** *for each part* based on timing for each operations. (Although tables/software are available, the most accurate numbers are compiled through time studies in particular factories.)

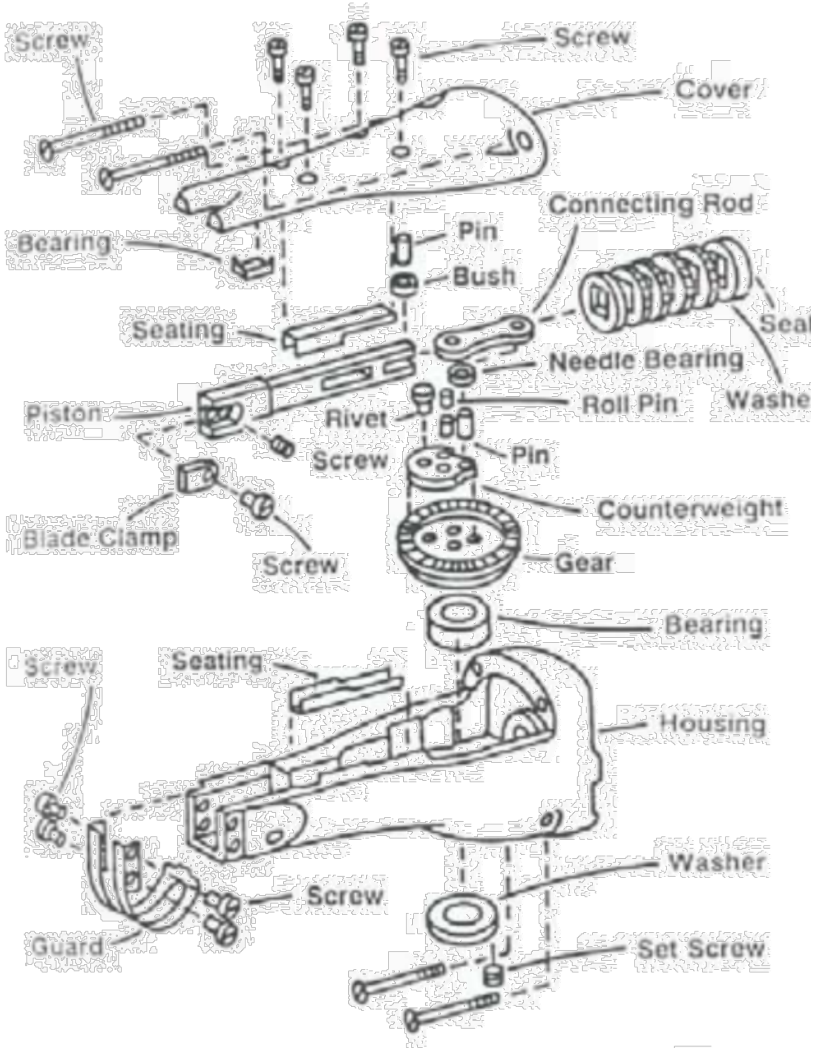
General Methodology

1. Select an **assembly method** for each part
2. **Analyze** the part for the given assembly methods
3. **Refine the design** in response to shortcomings identified by the analysis
4. *Loop to step 2 until the analysis yields a sufficient design*

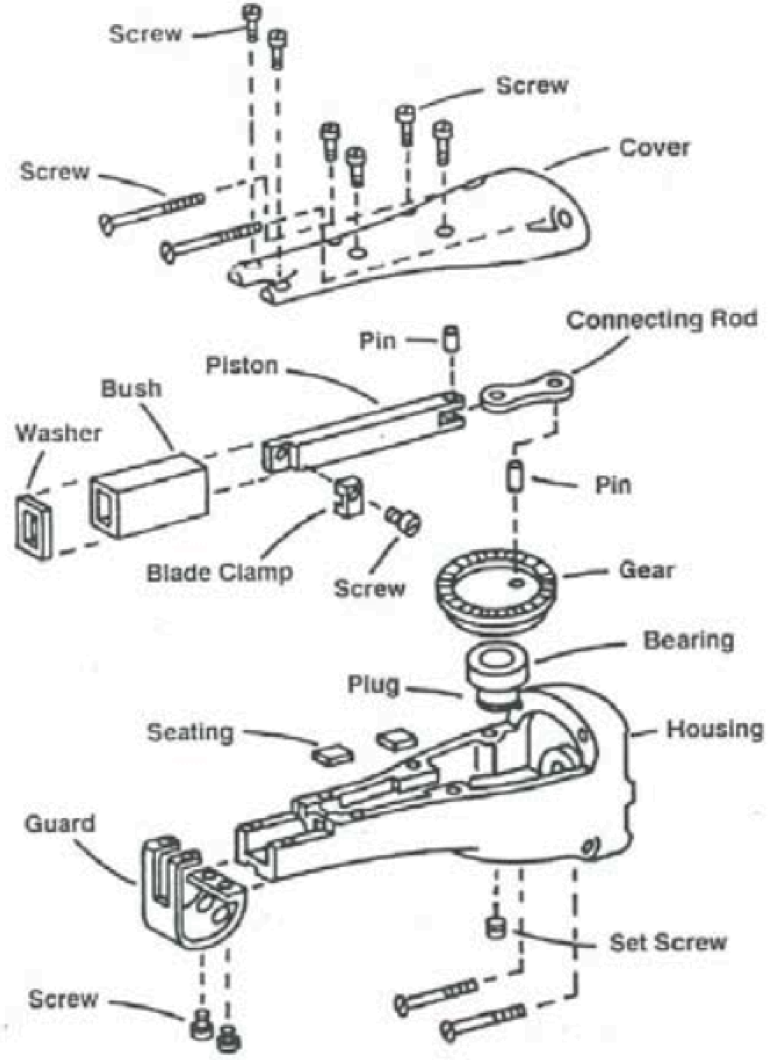
Illustration: Optimizing number of parts



Original test case (power saw)



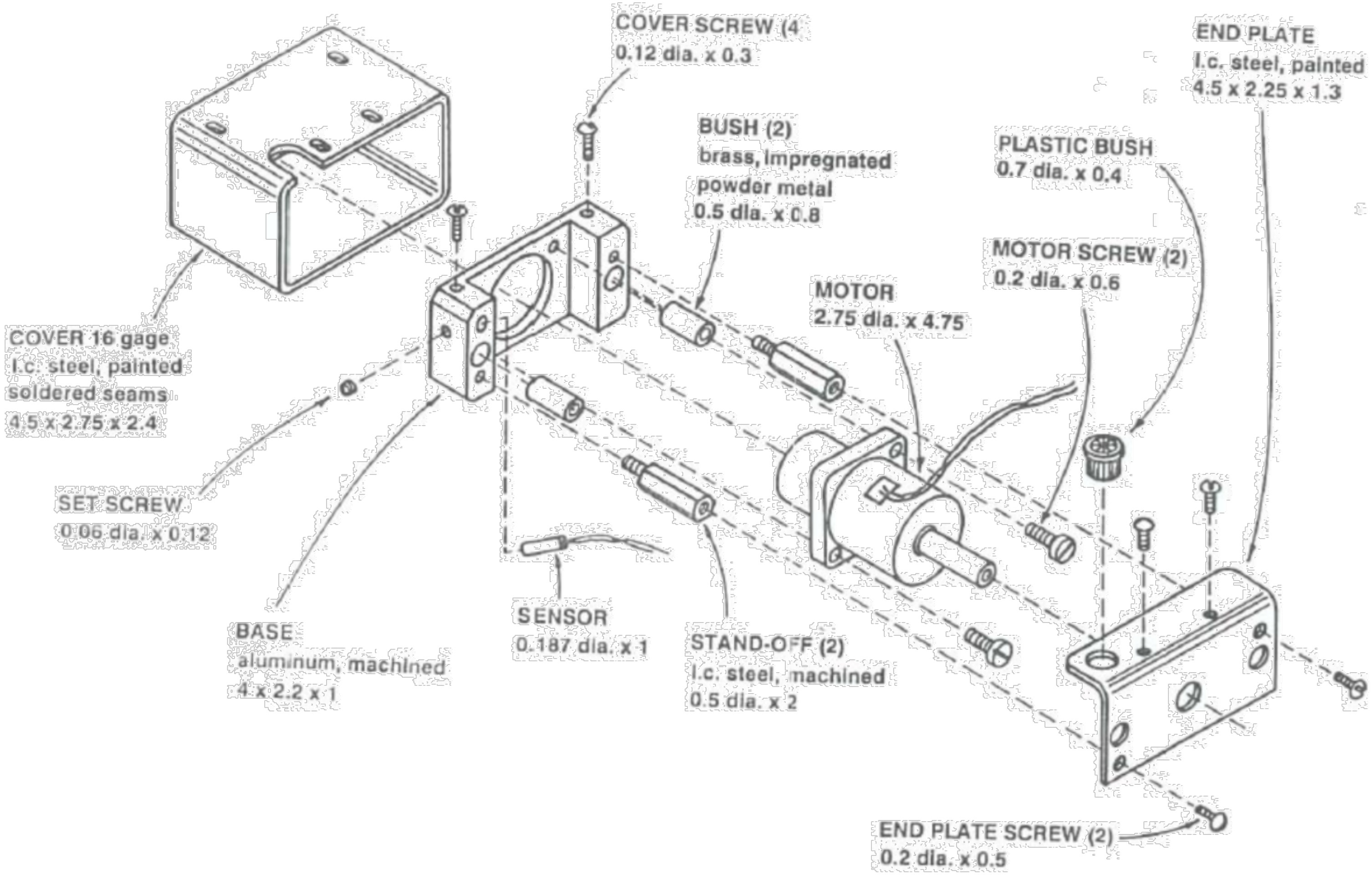
41 parts, t = 6.37 min



29 parts, t = 2.58 min

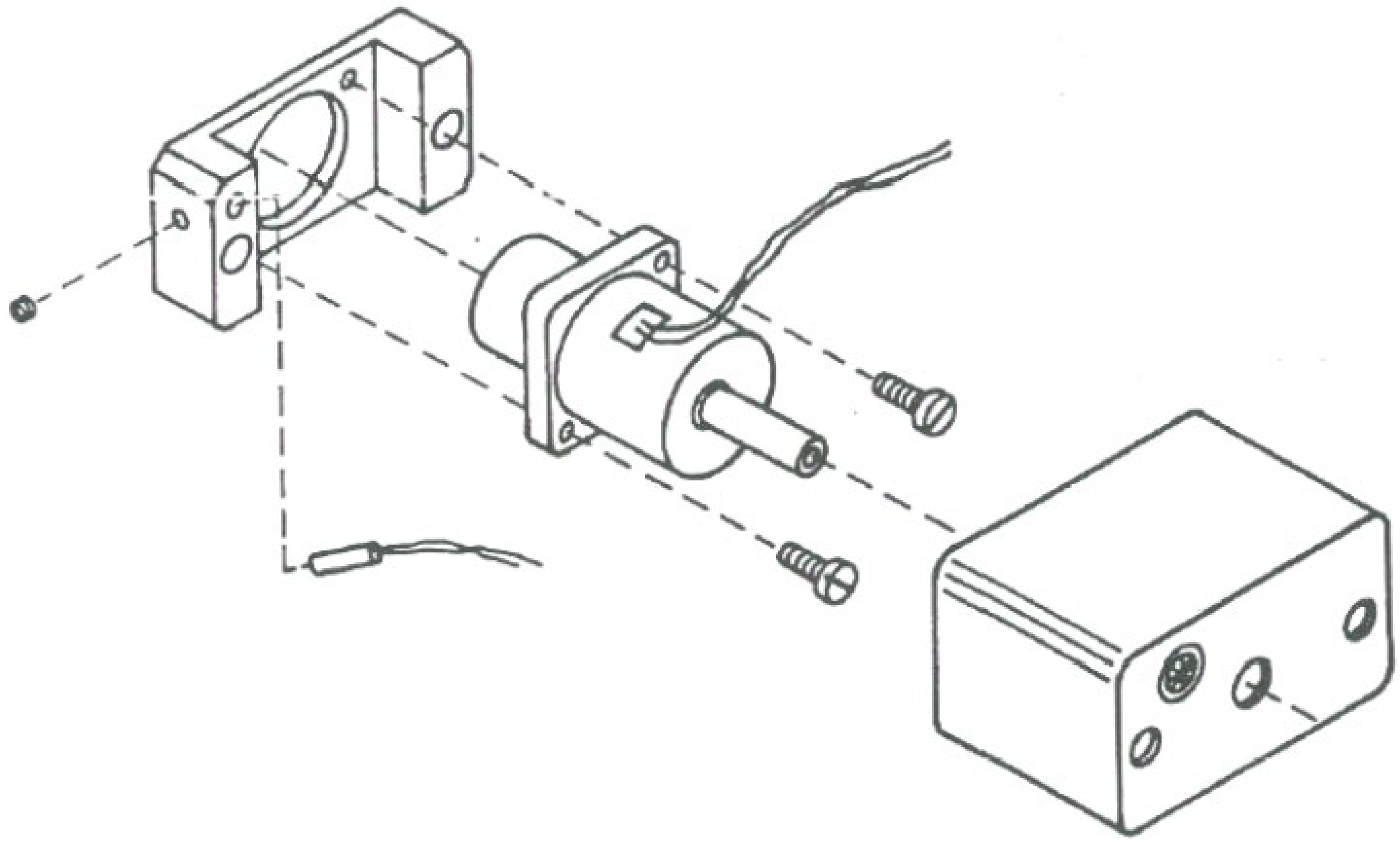
(source Boothroyd, Univ. of Mass.)

Example: Before DFMA



(Source: Boothroyd)

Example: after DFMA



(Source: Boothroyd)

Design for Assembly steps

Step 1
- Product Information: ***functional requirements***
- Functional analysis
- Identify parts that can be standardized
- Determine part count efficiencies

Step 2
- Determine your **practical** part count

Step 3
- Identify **quality** (mistake proofing) opportunities

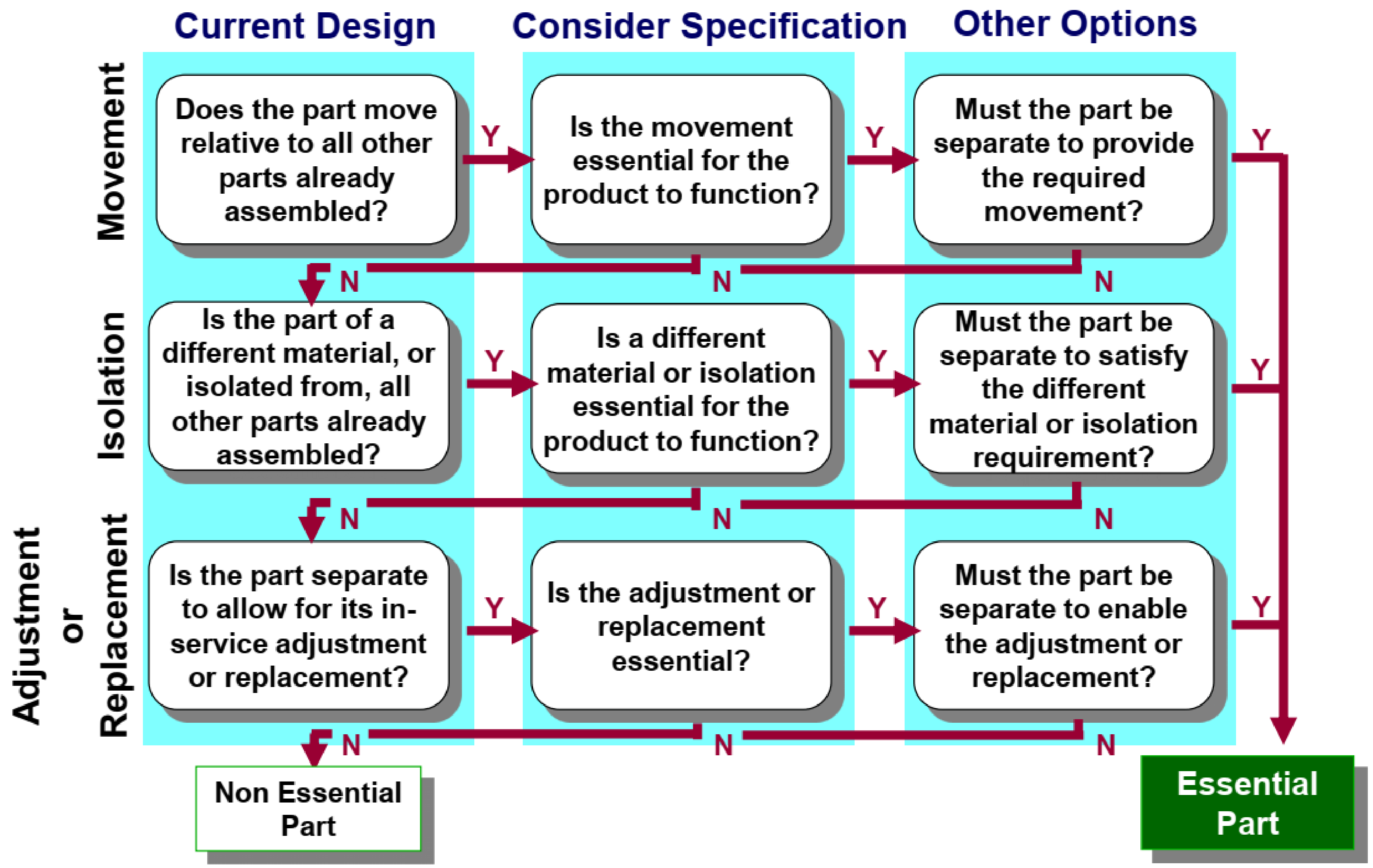
Step 4
- Identify **handling** (grasp & orientation) opportunities

Step 5
- Identify **insertion** (locate & secure) opportunities

Step 6
- Identify opportunities to reduce **secondary operations**

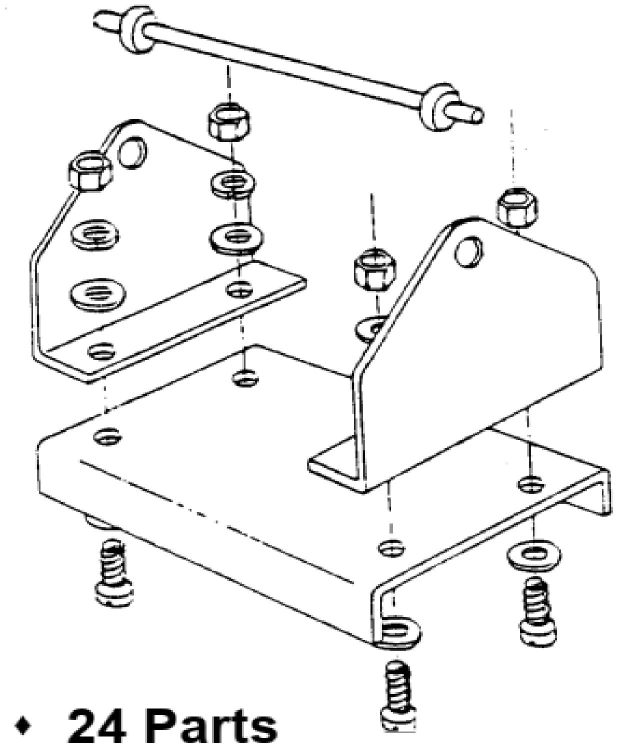
Step 7
- Analyze data for **new design**

Functional analysis: is a part essential?



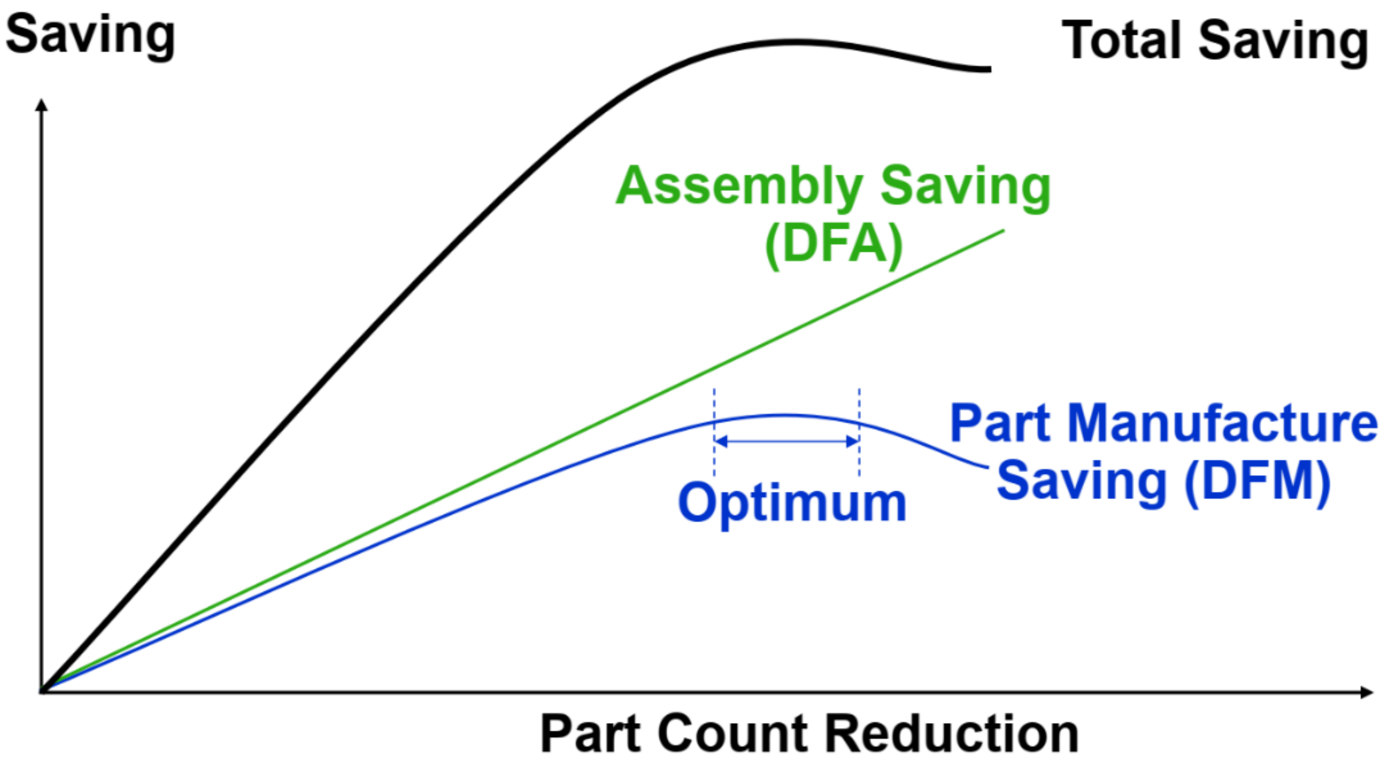
Source: David Stienstra (Rose-Hulman)

Illustrative exercise



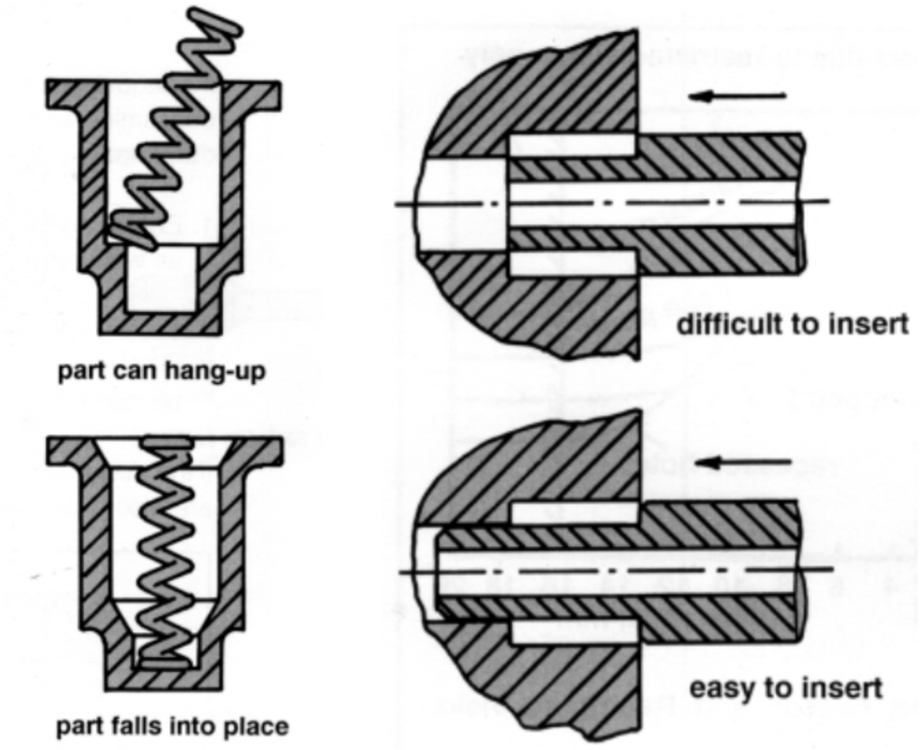
- 24 Parts
- 8 different parts
- multiple mfg. & assembly processes necessary

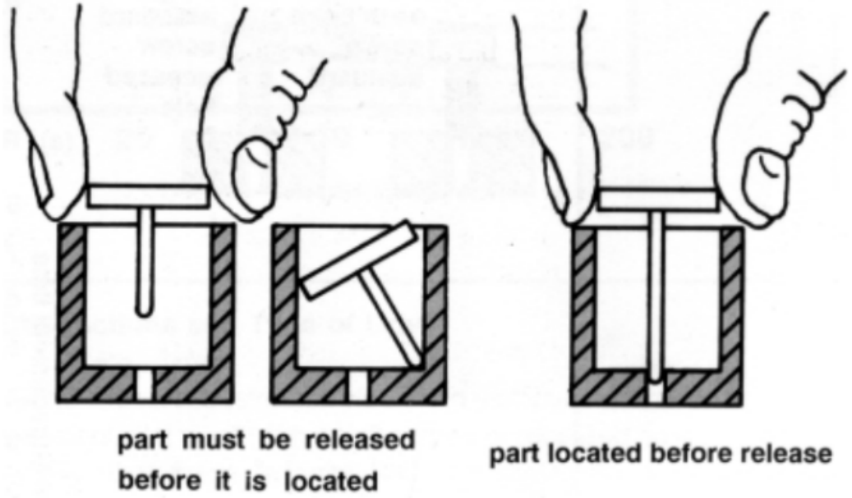
Reducing assembly complexity vs manufacturing complexity



Source: David Stienstra (Rose-Hulman)

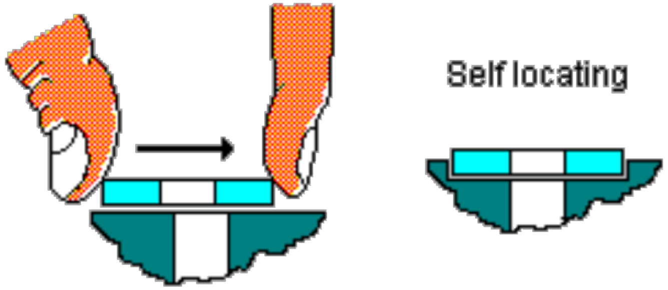
Further examples of analysis / refinement

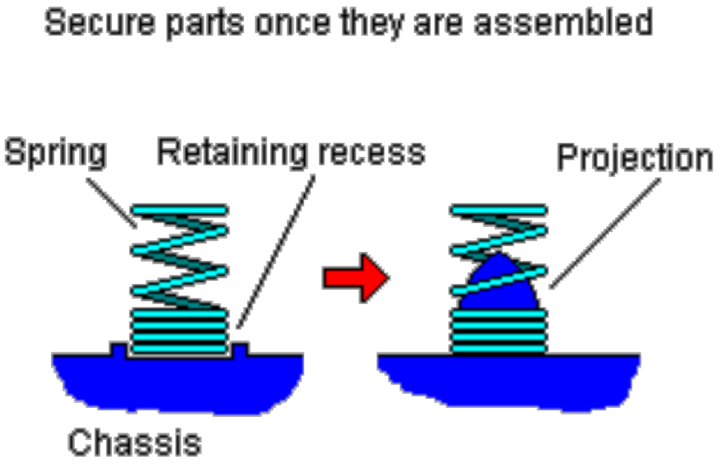


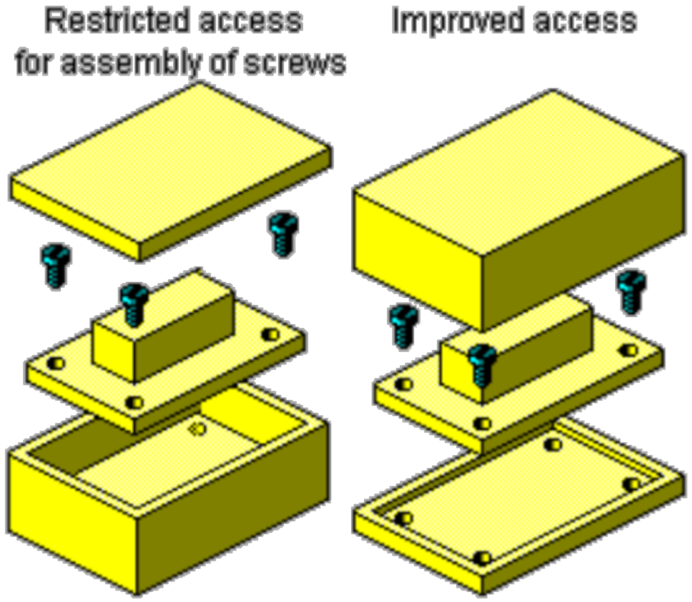


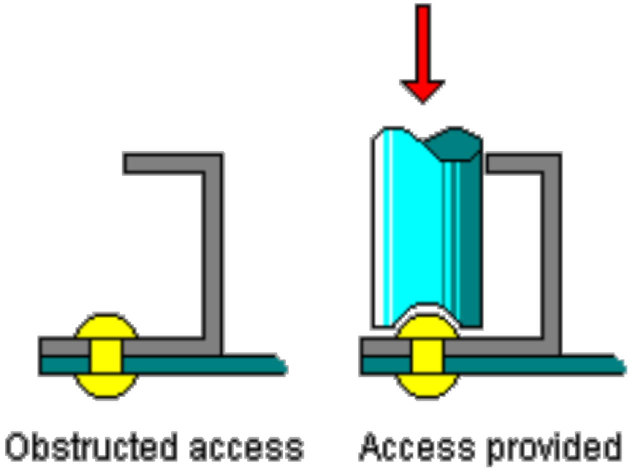
Source: Boothroyd & Dewhurst

Further examples of analysis / refinement









Source: David Stienstra (Rose-Hulman)

Case-study: DFM applied to precision assembly

Design for assembly **(DFA)**

'Design for assembly (DFA) is a process by which products are designed with ease of assembly in mind'

- A variety of **design rules** to:
  - Lower costs by reducing number of parts
  - Optimizing parts for simpler assembly
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- An ensemble of **trade-off**
  - **Fewer parts** vs **performance** / robustness
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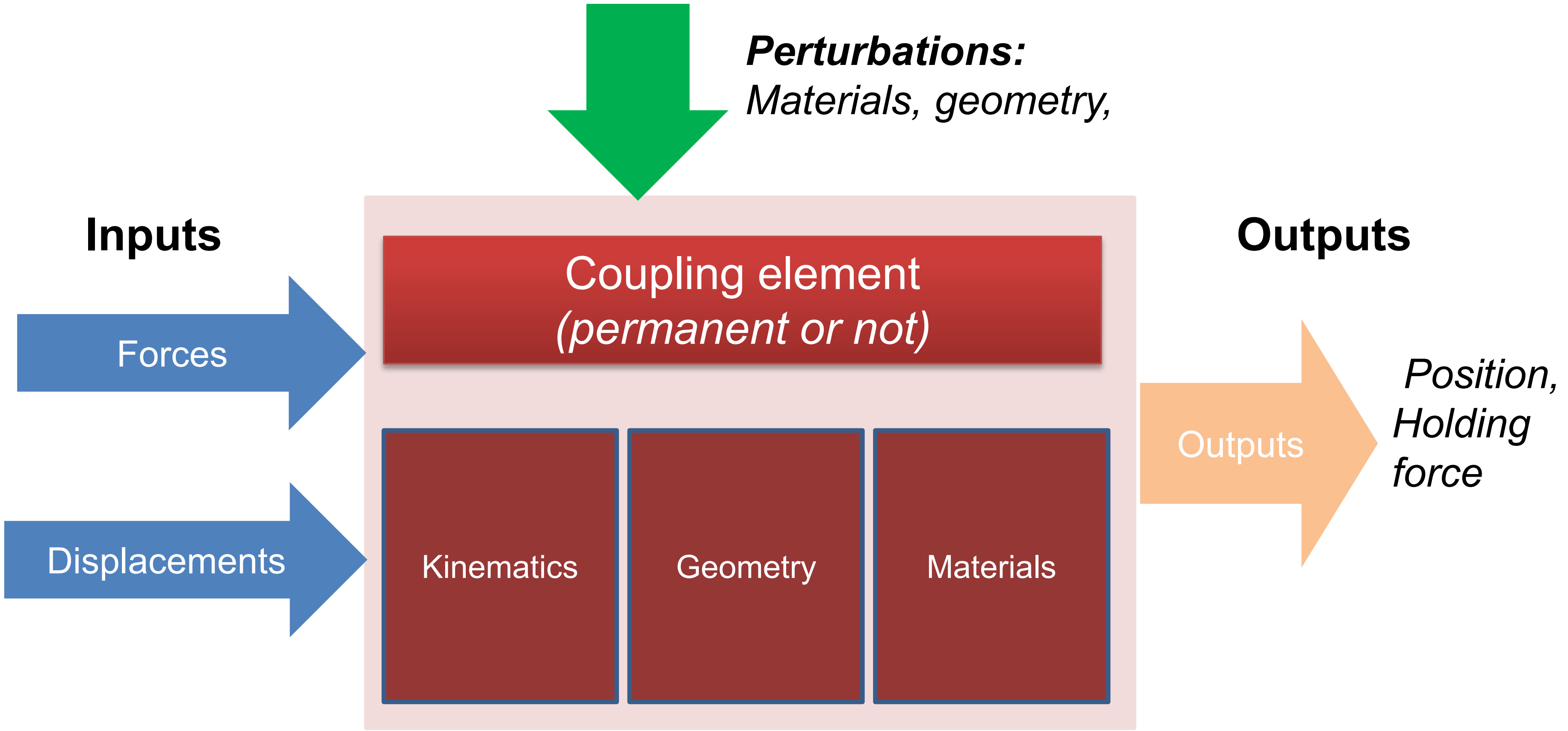
Specific on precision assembly: order of magnitude

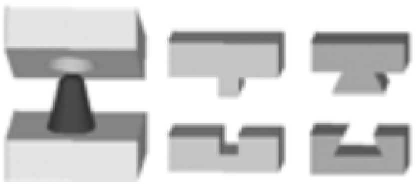
Applications	System Size	Highest precision requirements
Fiber optics / small optics	Meso- to micro-scale	‘Nano’ (<100 micron)
Optical resonators	Meso-, micro scale	Nano
Large array telescopes	Macro-scale	Angström
Cars	Few meters size	~ 1 micron

Assembly / couplings are **systems!**



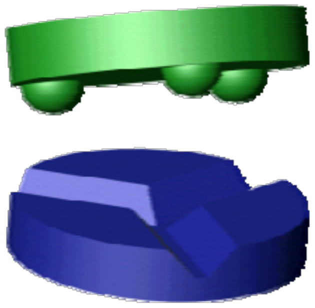
- Requires a **system design** approach!

Common couplings



Elastic Averaging

Non-Deterministic

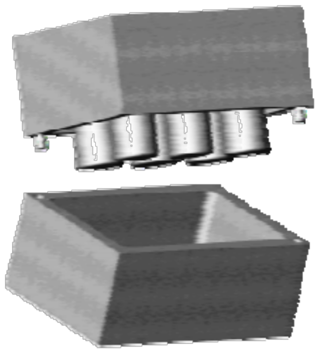


Kinematic Couplings

Kinematic Constraint

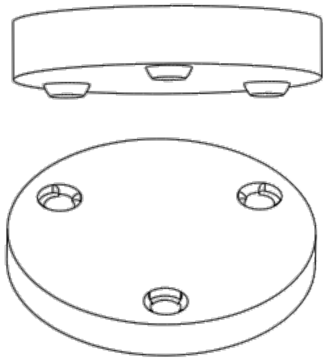
Flexural Kin. Couplings

Kinematic Constraint



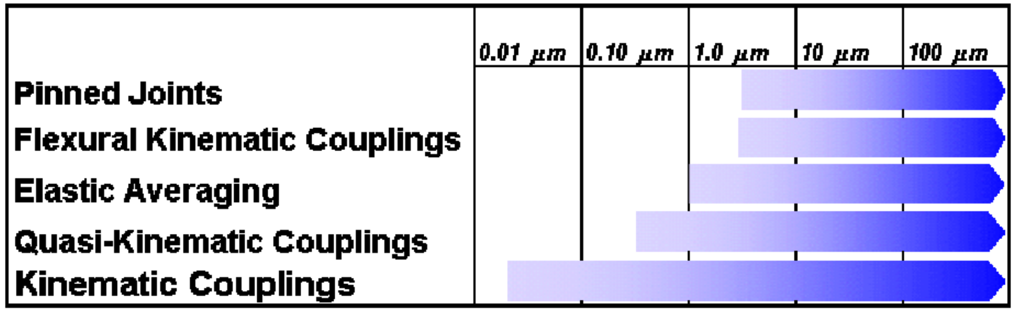
Pinned Joints

No Unique Position



Quasi-Kinematic Couplings

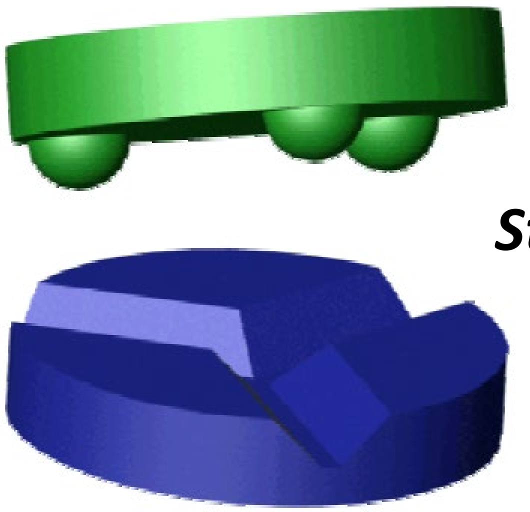
Near Kinematic Constraint



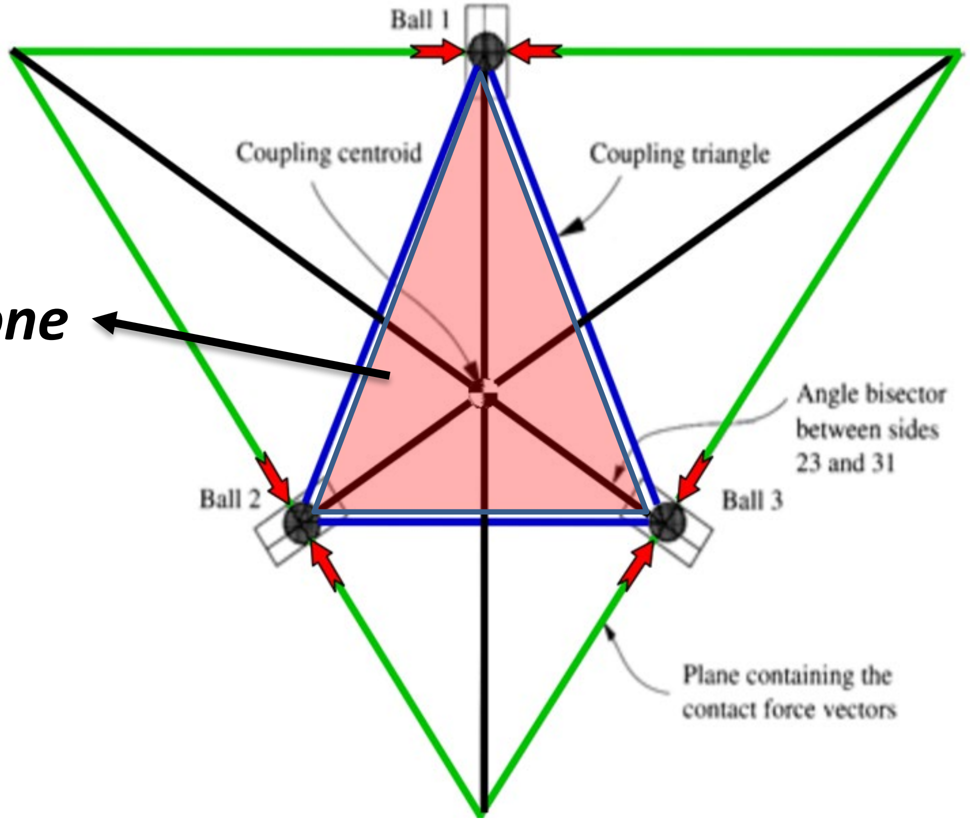
(Source MIT, Alex. Slocum)

Stability, positioning errors

- **Stability** is affected by *the balance of forces* applied to the coupling



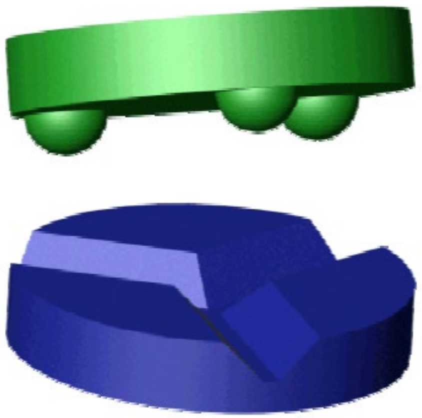
Stability zone

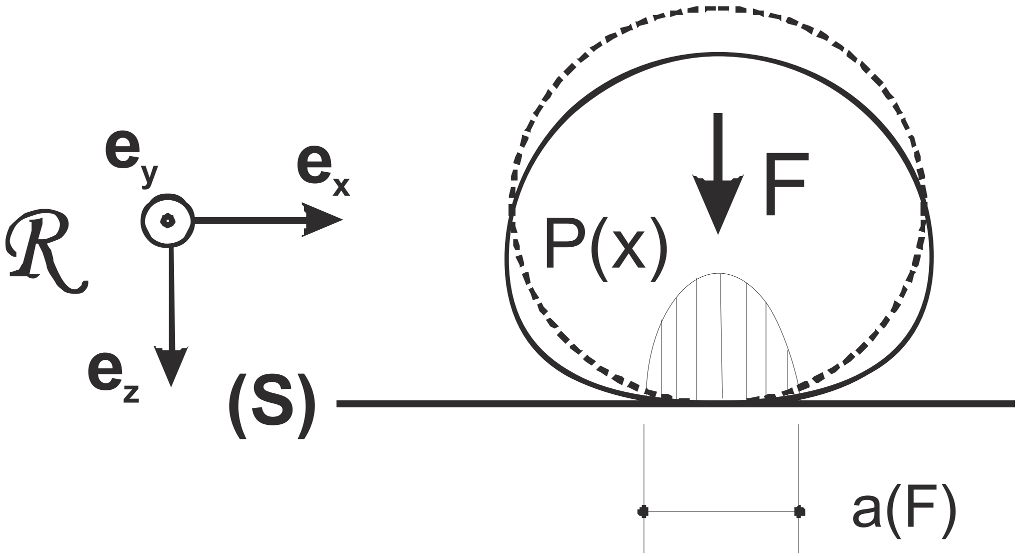


(Illustrations: MIT, Alex. Slocum)

Stability, **positioning errors**

- **Positioning errors** is affected by the *deformation* of contact points (Hertz contact pressure – see lecture 'surfaces')





Overconstraining or not?
(Hyperstatic vs statistically determinate)

- Depends on the function of the linkage:
  - **Stability/robustness**
    versus
  - **Precision/repeatability** of the positioning
- Isostatic systems leads to a predictable system (for instance: known force loop, uniquely defined positions, controlled thermal expansion behavior, ...)
- Hyperstatic implies the existence of internal stress

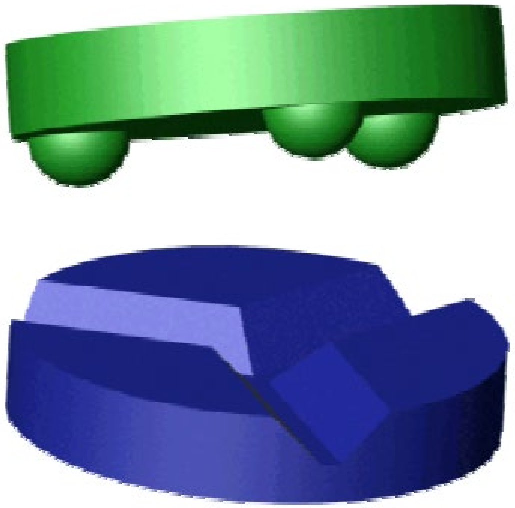
(source: 1stlibs.com)

Precision positioning

- **Theoretical goal** is to achieve exactly zero mobility (m=0), conversely, just as many constraints than degree of freedom. In other words: no **over-constraint!**

- **However:**
  - In practice, **full 'isostatic' case** is not possible and can only be achieved with a given resolution
  - As seen earlier, things move... During and possibly after assembly (difference in CTEs, creep effect, etc.)
  - Cost is **inversely proportional** to the level precision required

Discussion

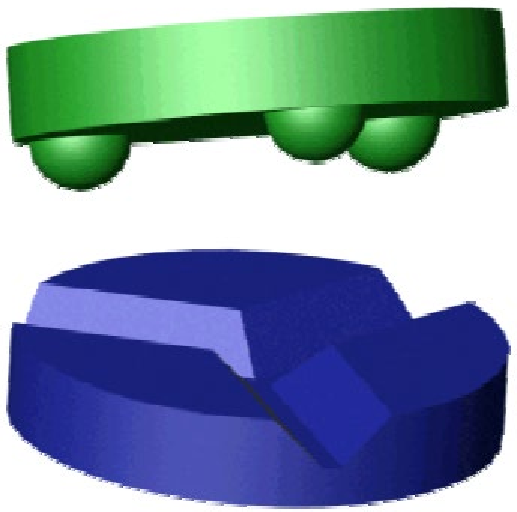


- How would you make such part in practice?

Constraints:

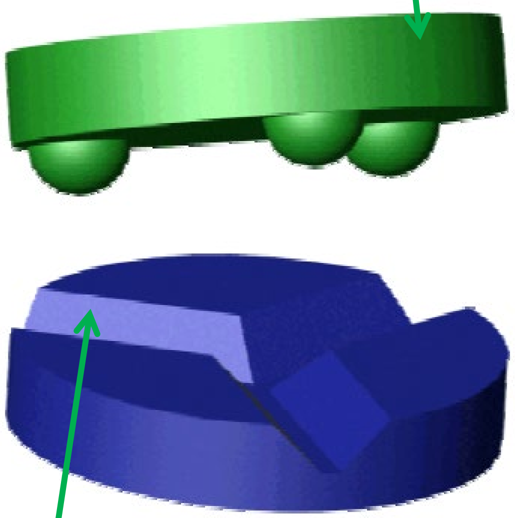
- Easy-to-make
- Fulfilling isostatic conditions
- Can be produced in large quantities

From a concept to an actual implementation: design for manufacturing and assembly

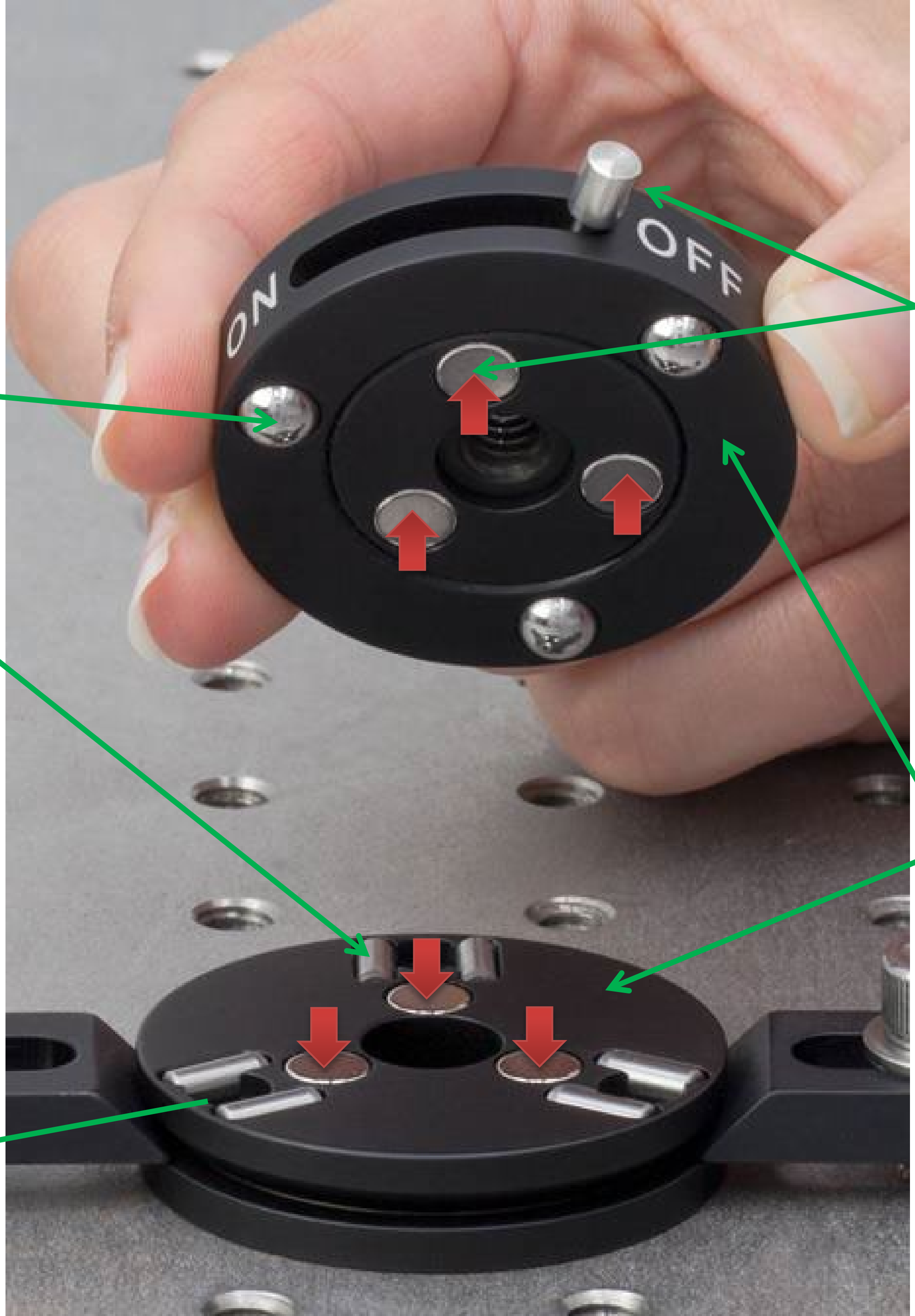


Discussion

Steel is only used where needed and in simple to manufacture shapes (spheres, pins)

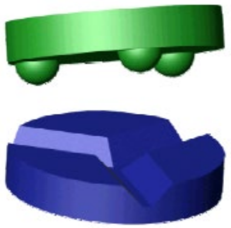


Two-pins cylinders are equivalent to a V-groove and a sphere contact



Magnets are used for exerting a constant attraction force (when they are aligned with their counter-parts). They can be rotated to 'activate' the force.

An aluminum (easy-to-manufacture, cost-effective) is used for the main structure where there is no high contact-pressure



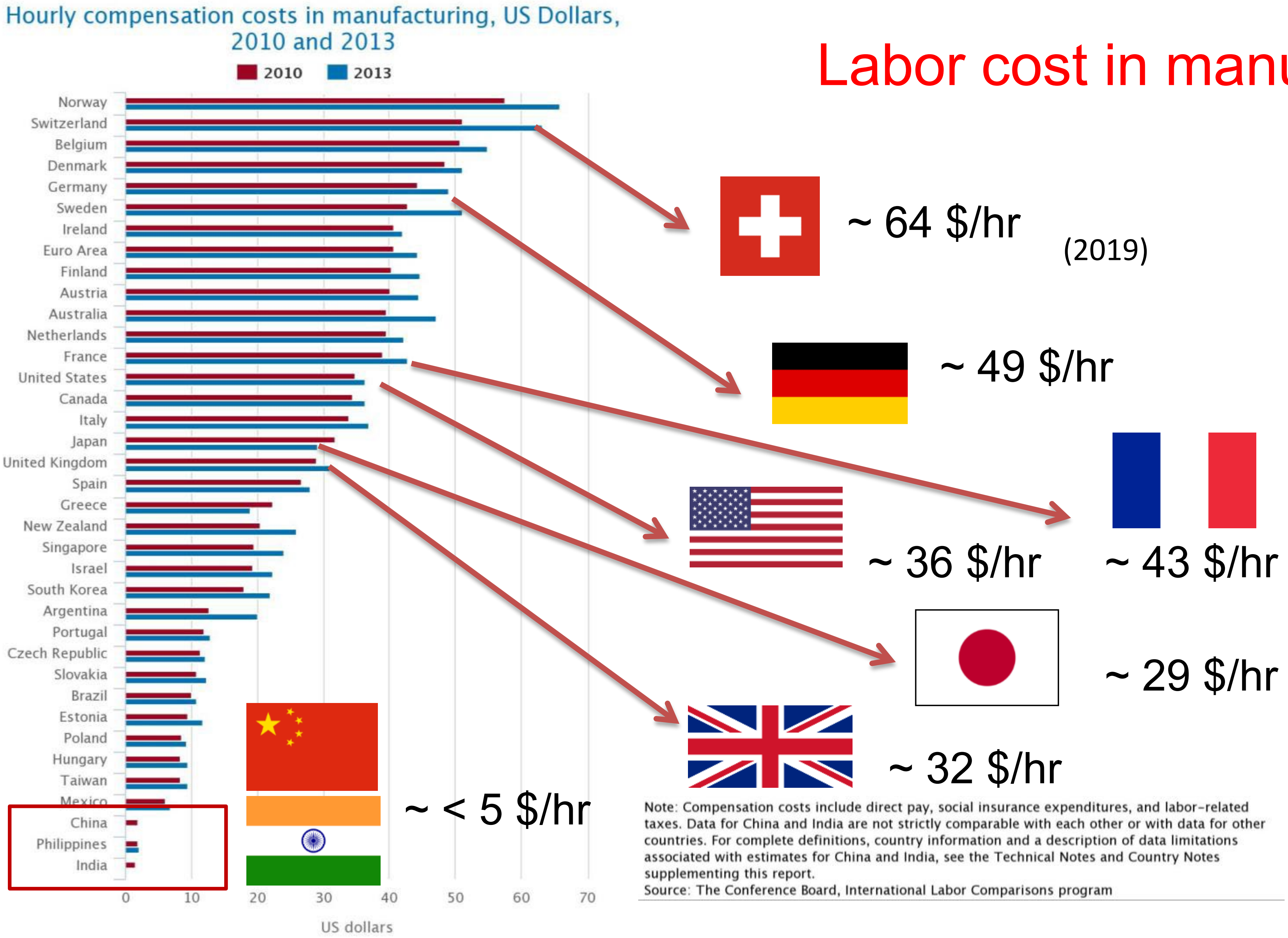
Assembly costing model

- Two main terms, handling and fitting operations expressed in sec.

$$C_{ma} = C_L \left(F + H \right)$$

- C_{ma}: Cost for manual assembly
- C_L: Labor cost (CHF/s)
- F: Component fitting index **(seconds)**
- H: Component handling index **(seconds)**

Labor cost in manufacturing...

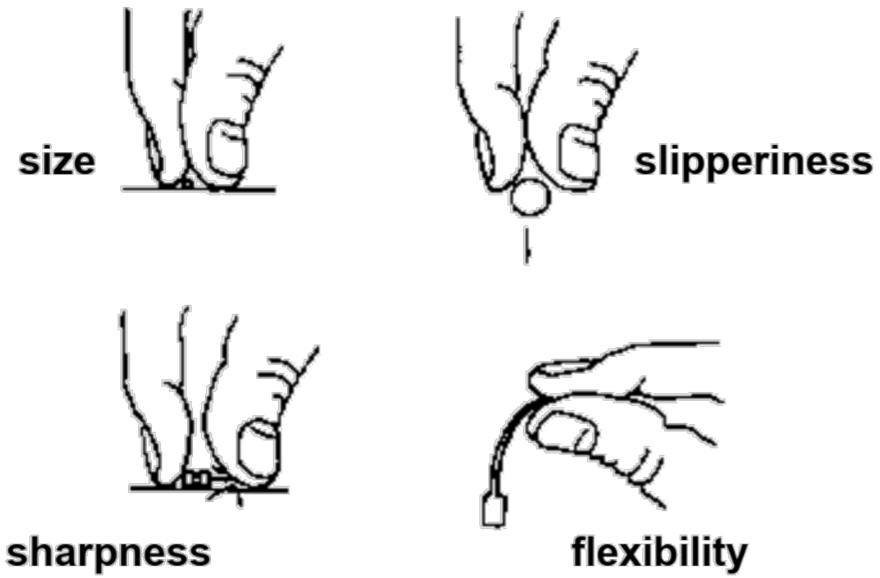


Note: Compensation costs include direct pay, social insurance expenditures, and labor-related taxes. Data for China and India are not strictly comparable with each other or with data for other countries. For complete definitions, country information and a description of data limitations associated with estimates for China and India, see the Technical Notes and Country Notes supplementing this report.
Source: The Conference Board, International Labor Comparisons program

Source: WTO

Handling difficulties...

- Size
- Thickness
- Weight
- Flexibility
- Slipperiness
- Stickiness
- Necessity of assistance (two hands, optical magnification, etc.)



→ *Idea is to define a basic handling index ('cost function')*

Component Handling Analysis

- Definition

$$H = A_h + \left[\sum_{i=1}^{n} P_{O_i} + \sum_{i=1}^{n} P_{G_i} \right]$$

Total cost linked to handling parts (H)

Basic handling index for an ideal design using a given handling process (A_h)

Orientation Penalty for the component design (P_{O_i})

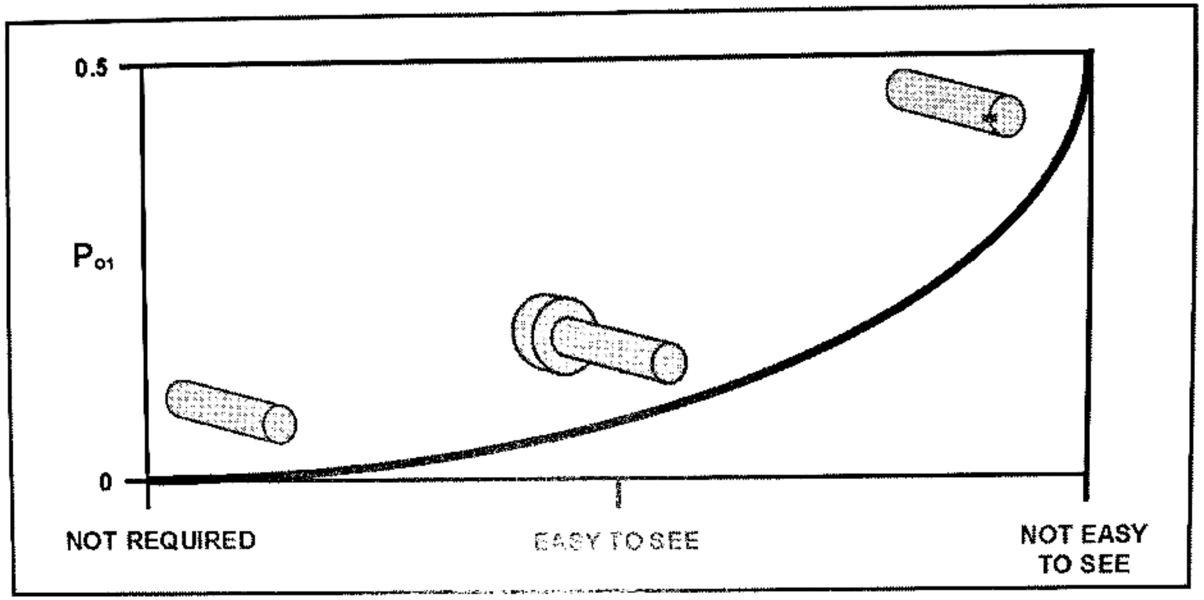
General handling property penalty (P_{G_i})

Basic handling index (A_H)

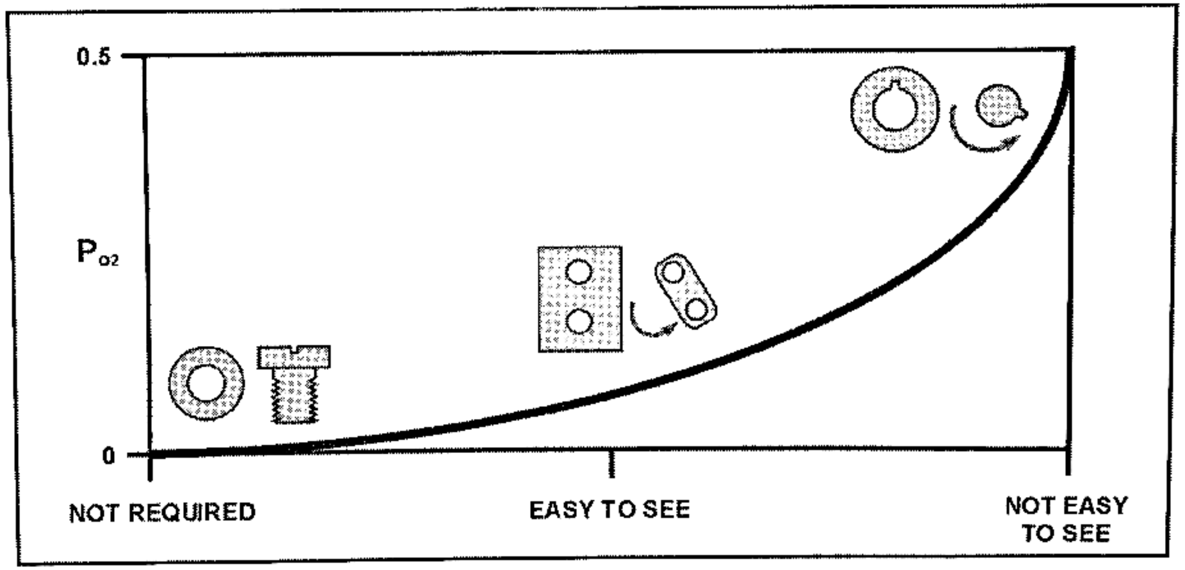
Component handling characteristic	Index (Ah)
One hand only	1
Very small aids / tools	1.5
Large and/or heavy (two hands/tools)	1.5
Very large and/or very heavy (two people/hoist)	3

Orientation penalties

End-to-End orientation (along axis of insertion)



Rotational Orientation (about axis of insertion)



Handling Sensitivity Index (P_g)

Component handling sensitivity	Index (Pg)
Fragile	0.4
Flexible	0.6
Adherent	0.5
Tangle/severely tangle	0.8/1.5
Severely nest	0.7
Sharp / abrasive	0.3
Hot/Contaminated	0.5
Thin (gripping problem)	0.2
None of the above	0

Component 'Fitting' analysis

- Definition

$$F = A_f + \left[\sum_{i=1}^{n} P_{f_i} + \sum_{i=1}^{n} P_{a_i} \right]$$

Basic fitting index for an ideal design using a given assembly process

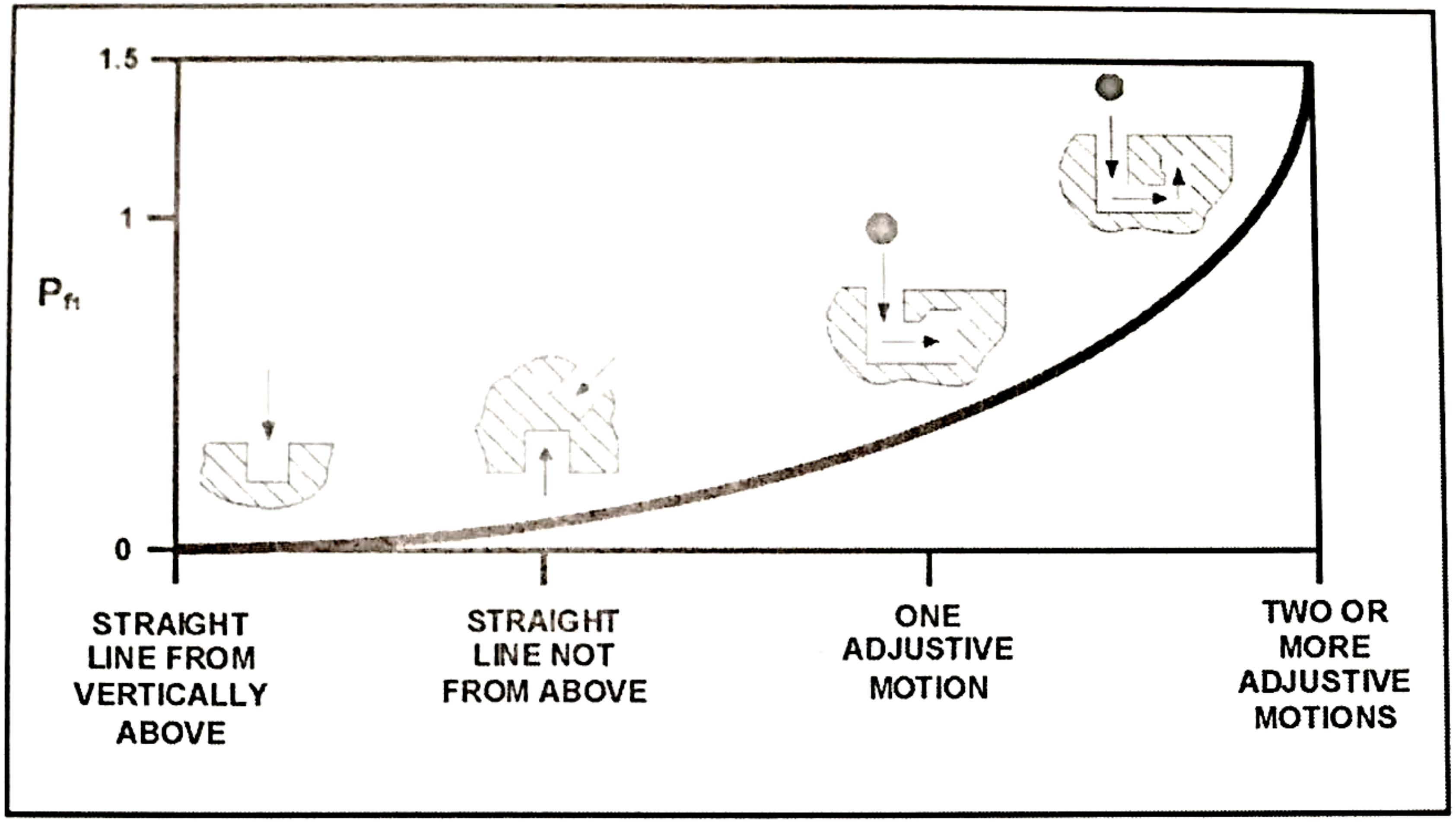
Insertion penalty

Penalty for additional processes on parts in place

Basic Component Fitting index (A_F)

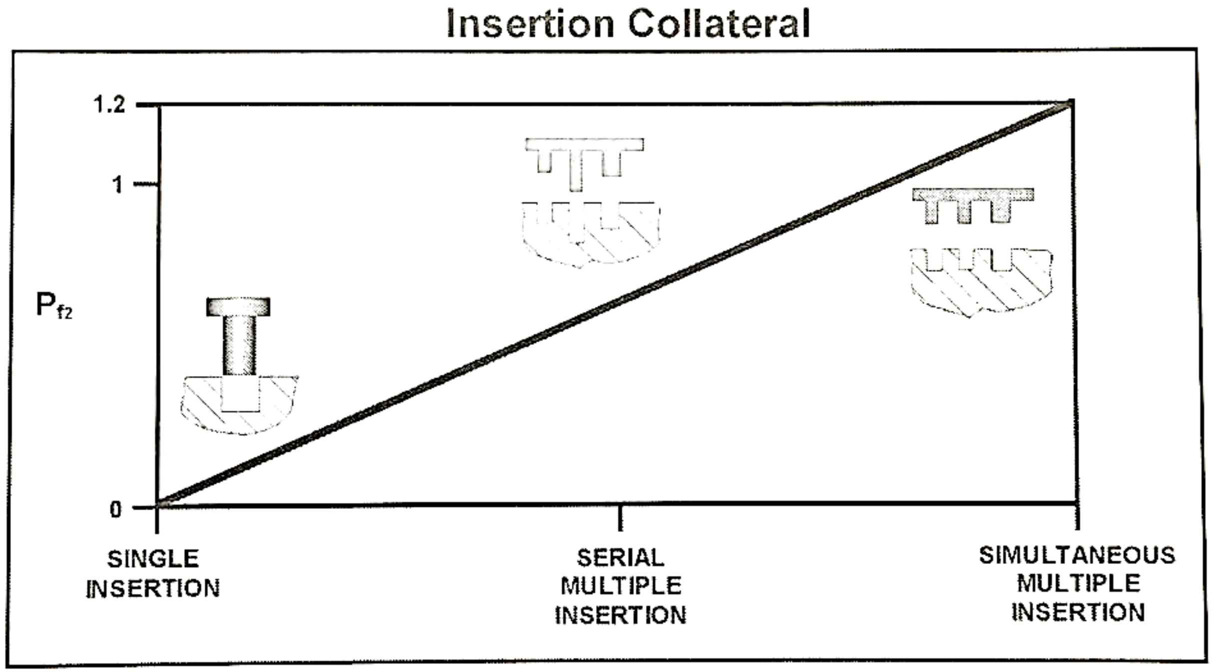
Assembly process	Index (Af)
Insertion only	1
Snap fit	1.3
Screw fastener	4
Rivet fastener	2.5
Clip fastener (plastic bending)	3

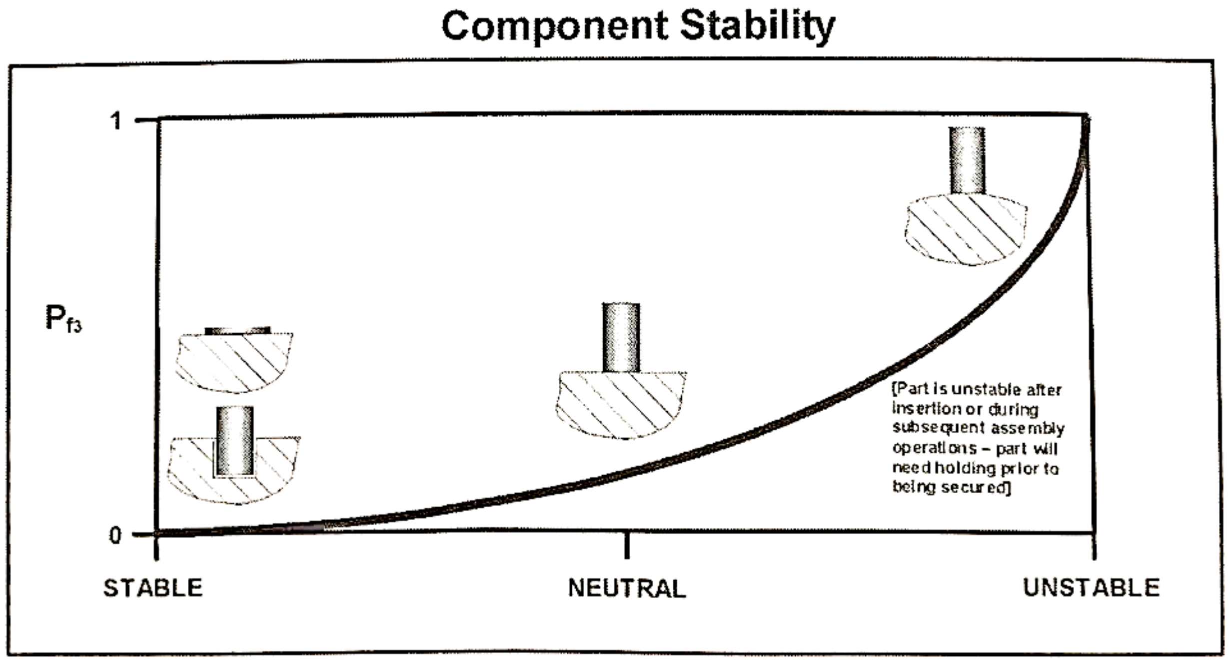
Insertion direction penalty



(source: Swift, Booker, ‘Manufacturing process selection handbook’, Butterworth-Heinemann)

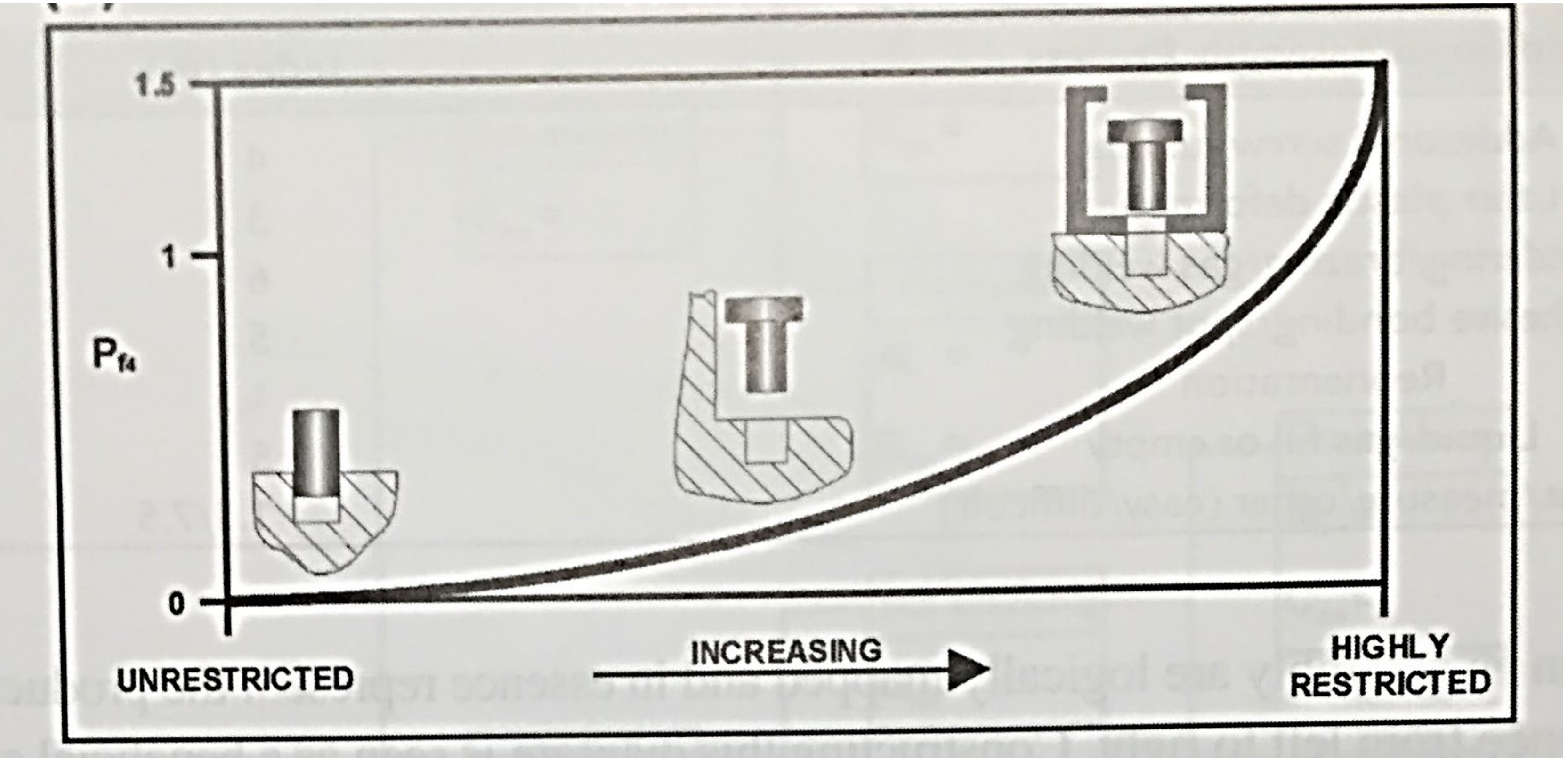
Insertion collateral and stability





(source: Swift, Booker, 'Manufacturing process selection handbook', Butterworth-Heinemann)

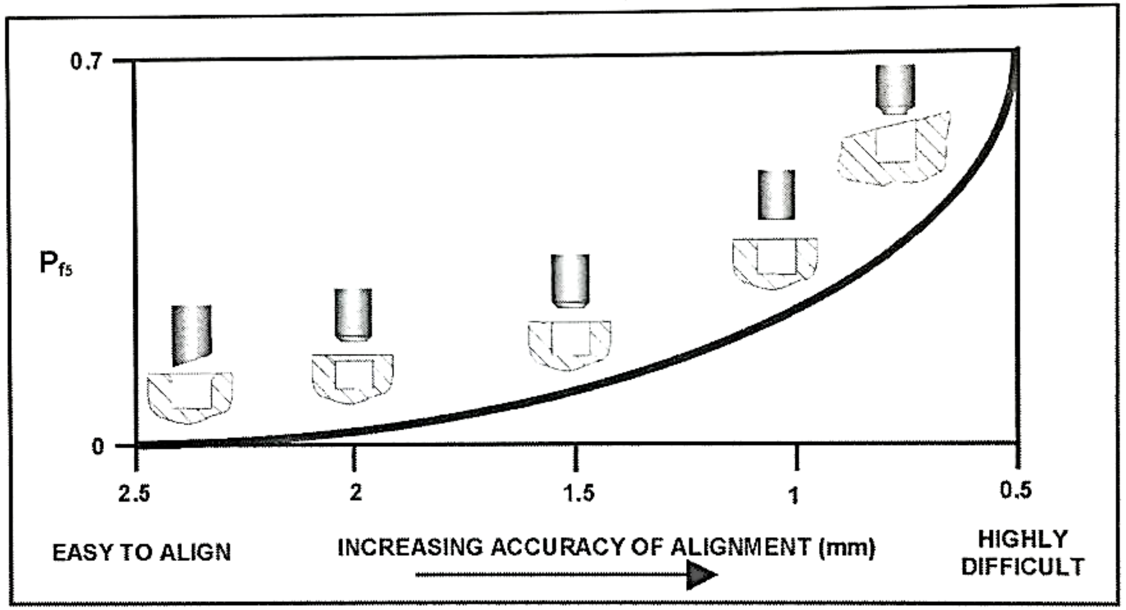
Insertion penalties



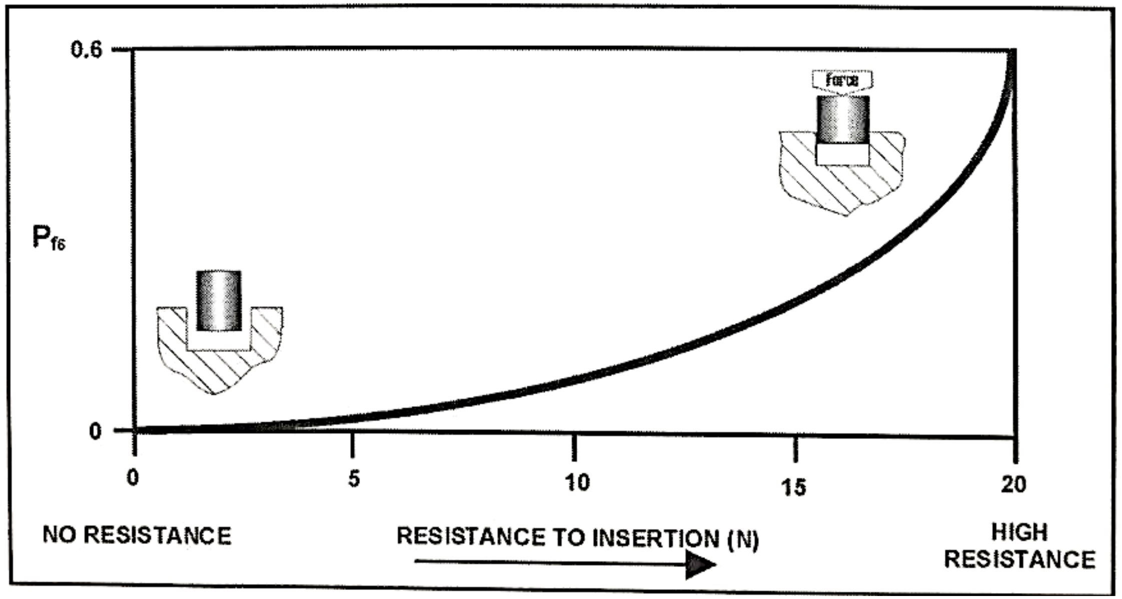
(source: Swift, Booker, 'Manufacturing process selection handbook', Butterworth-Heinemann)

Insertion penalties

Insertion Alignment



Insertion Resistance

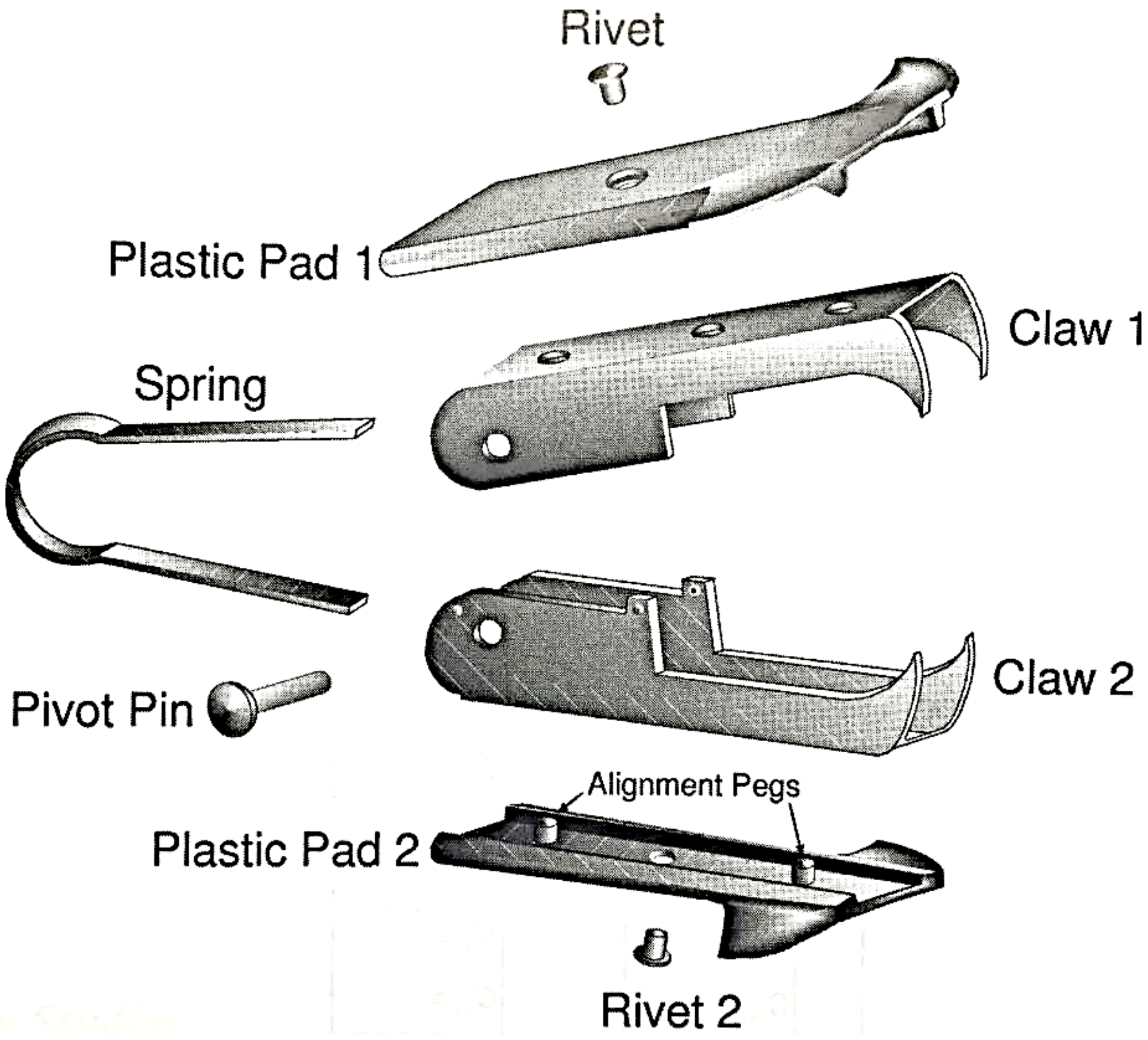


(source: Swift, Booker, ‘Manufacturing process selection handbook’, Butterworth-Heinemann)

Additional Assembly Index (Pa)

Additional Assembly process	Index (Pa)
Soldering/brazing/gas welding	6
Adhesive bonding/spot welding	5
Reorientation	1.5
Liquid/gas fill or empty	5
Set/test/measure/other...	1.5-7.5

Exercise: staple remover



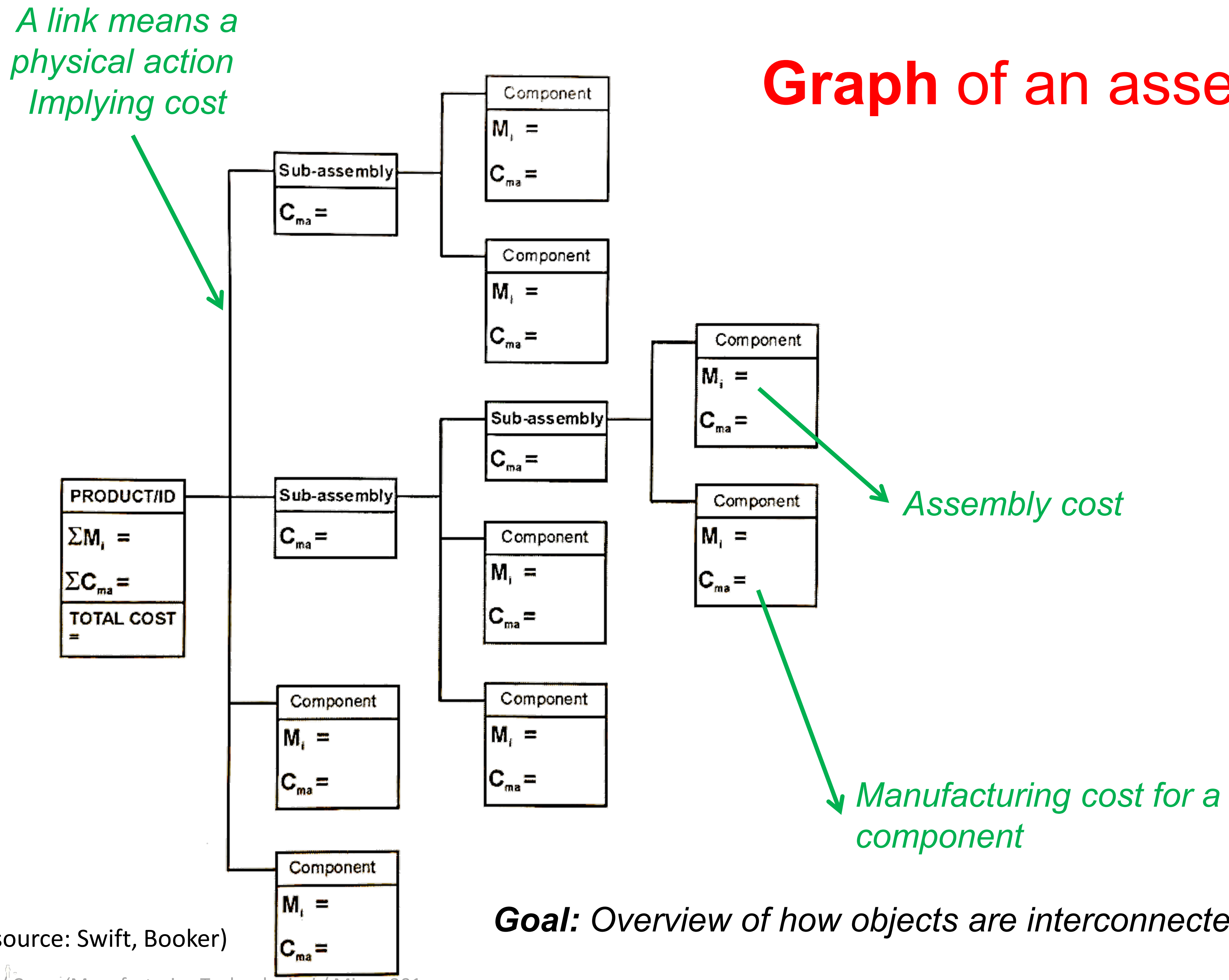
1 – Graph of the assembly
2 – Assembly cost estimate?

(source: Swift, Booker, 'Manufacturing process selection handbook', BH)

Methodology

1. Identify the individual parts
2. Identify how parts are linked together and sub-assemblies chains
3. Make a graph of the assembly
4. Identify handling and fitting operations for each components
5. Calculate the assembly cost

Graph of an assembly cost



(source: Swift, Booker)

Goal: *Overview of how objects are interconnected*

Illustration of a cost table for manual assembly

Assembly cost analysis table — **Stapler remover**

Labor rate **0.020** CHF/s — 71.2 CHF/h

Components assembly details				Handling operation analysis (H)						Fitting operation analysis (F)												Cost Assembly (in CHF)
Part ref.	Sub-assembly ref	Part description / sub-assembly desc.	Assembly process	Ah	Po1	Po2	Σ Po	Pg	Total Handing	Af	Pf1	Pf2	Pf3	Pf4	Pf5	Pf6	Σ Pf	Pa	Total Fitting		Total (F+H)	
1		Rivet 1	Hand./Fit.							2.5											4.1	fr. 0.08
2		Plastic pad 1	Hand./Fit.							1											2.2	fr. 0.04
3		Claw 1	Hand./Fit.							1											2.1	fr. 0.04
4		Rive6 2	Hand./Fit.							2.5											4.1	fr. 0.08
5		Plastic pad 2	Hand./Fit.							1											2.2	fr. 0.04
6		Claw 2	Hand./Fit.							1											2.3	fr. 0.05
	A	#1+#2+#1	Hand./Fit.							1											2.2	fr. 0.04
	B	#4+#5+#6	Hand./Fit.							1											3.7	fr. 0.07
7		Spring	Hand./Fit.							1											2.6	fr. 0.05
8		Pivot pin	Hand./Fit.							1											5.1	fr. 0.10
																					Total cost	**fr. 0.61**

Basic handling index

Basic fitting index

Additional assembly process

Individual components descriptions

Handling operations

Fitting operations

Orientation penalties

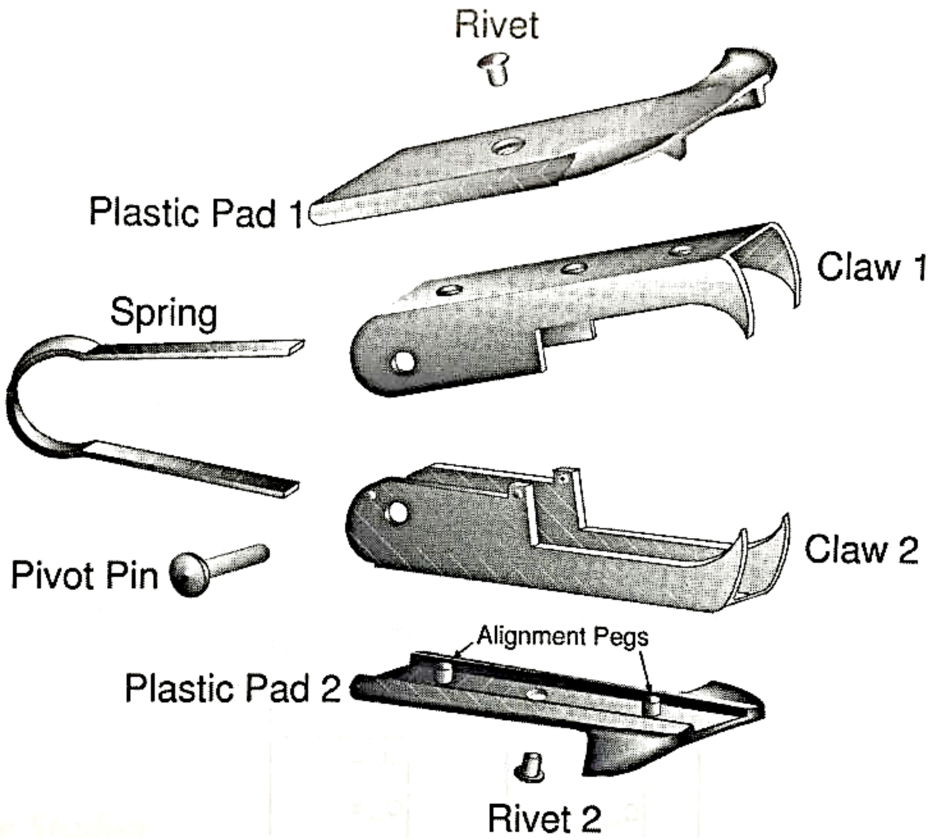
Handling sensitivity index

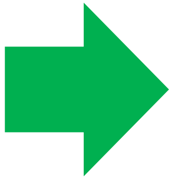
Insertion penalties

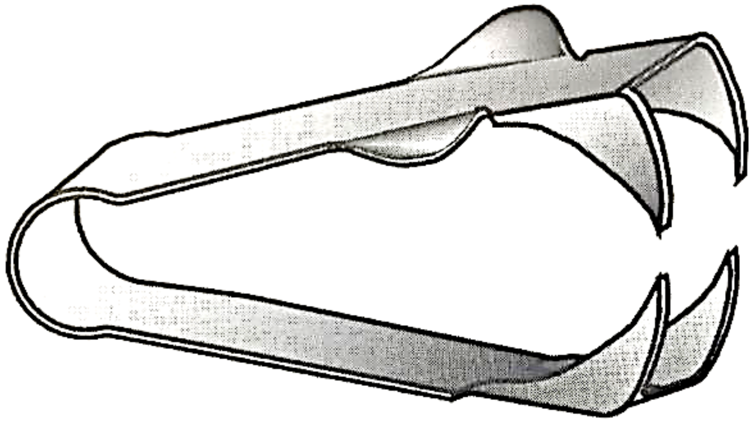
Assembly cost

$$C_{ma} = C_L \left(F + H\right)$$

Simplified, better design(s)?
(Design for assembly/manufacturing analysis)



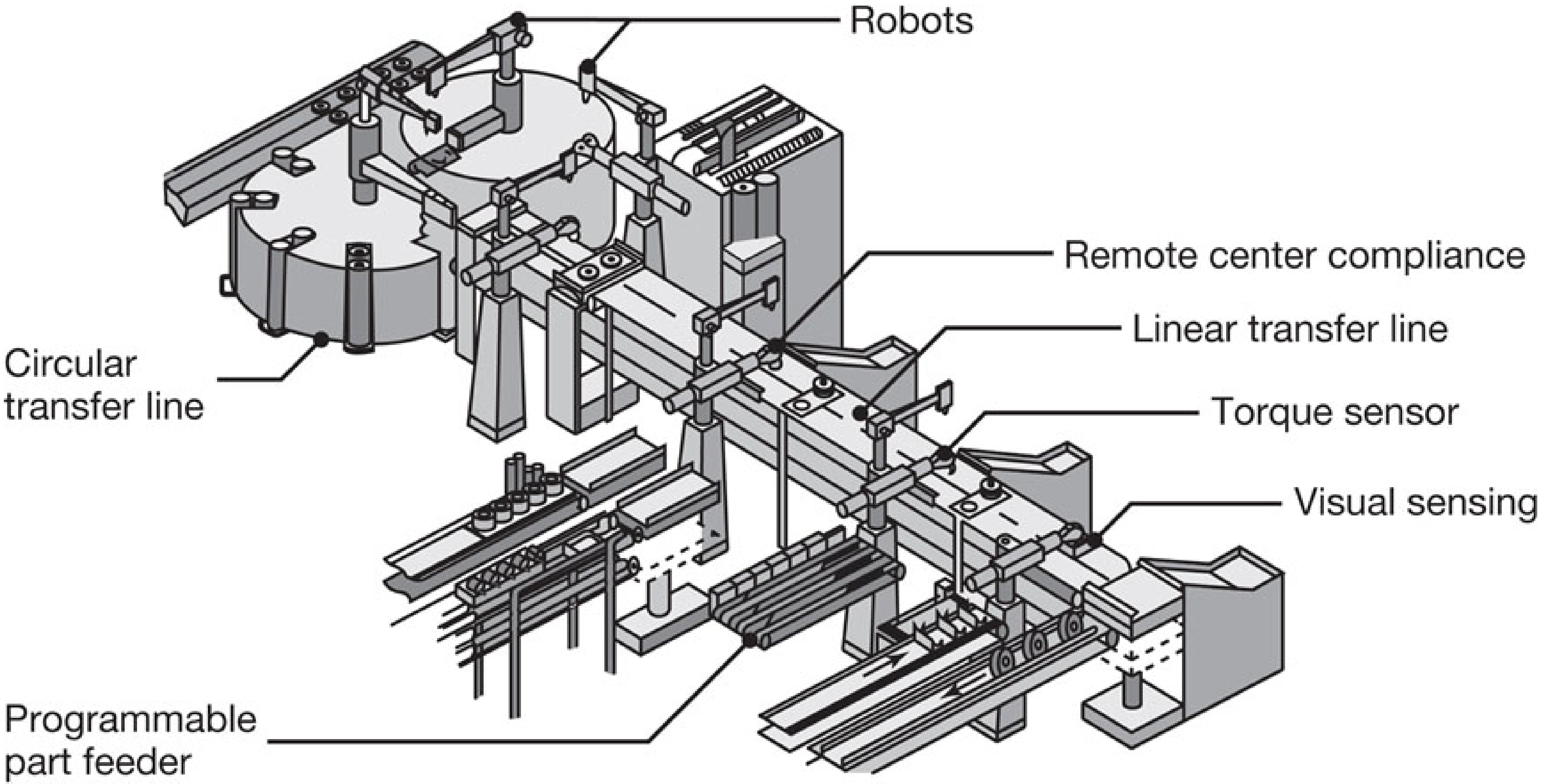




From manual to automated assembly...

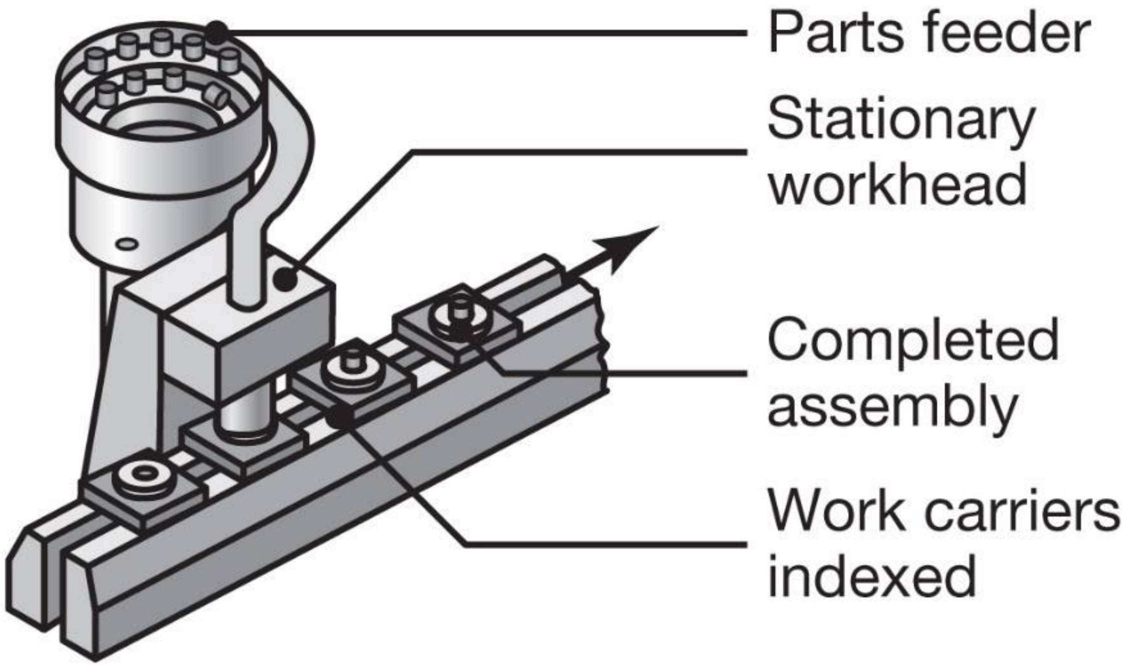
- Choice is driven by multiple considerations:
  - Production volume
  - Investment costs in automation
  - Necessity (tolerances requirements, sizes, etc.)
  - Labor costs
  - Production flexibility

A typical product assembly lines



(Illustration Kalpakjian, Schmid)

Transfer systems for automated assembly: Bowl feeder

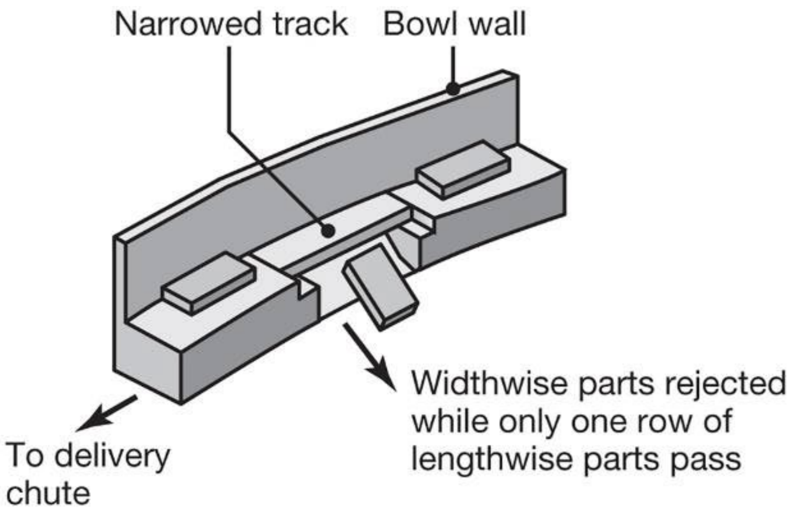


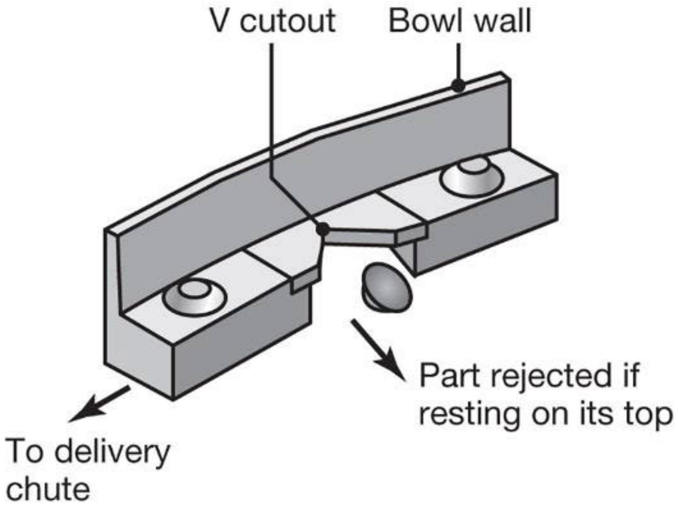
(Illustration Kalpakjian, Schmid)

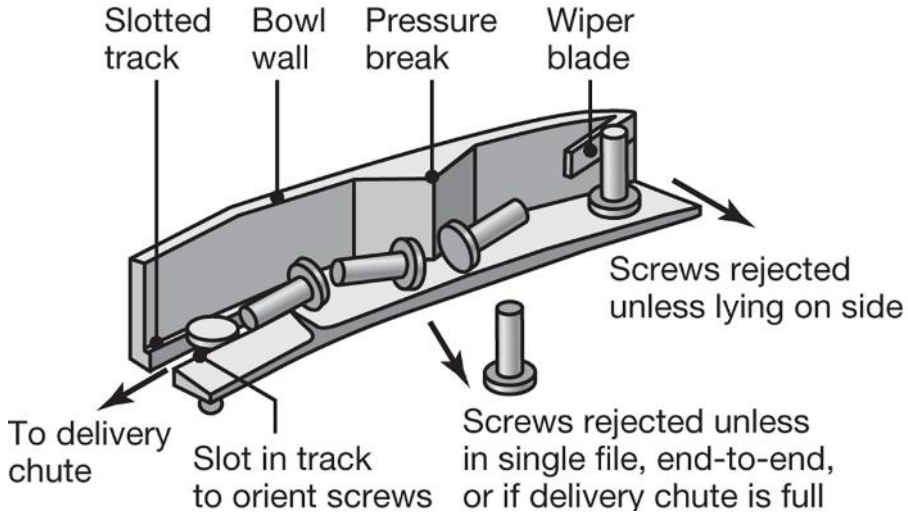
(Source: Wikipedia / public domain)

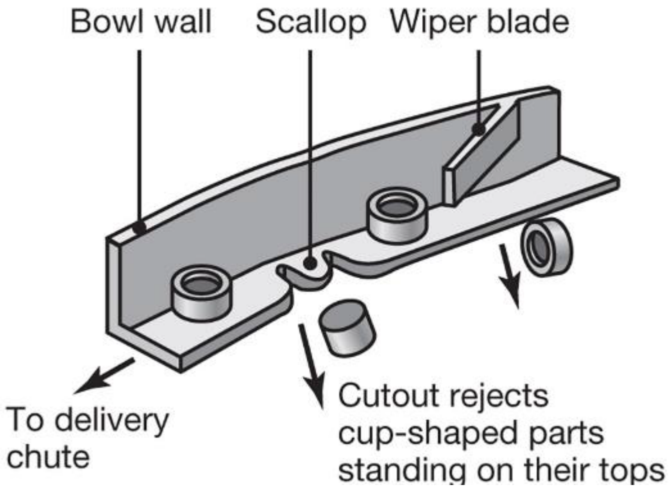
Working principle: *use vibrations and/or rotary motions to move unsorted path along a path where self-orientation sorting takes place.*

Bowl feeder: traps









(Illustration Kalpakjian, Schmid)

Bowl feeder: Additional videos

- Illustrative videos:
  - Research work on bowl feeder: https://youtu.be/BaQbX6-35Ho

- Other concepts (Asyril CH): ’shaker’+vision+pick-and-place
  - https://youtu.be/5GQSvhsgDCE

Joining materials

(Discussion in class)

- What does ‘**joining two parts**’ means? (in quantifiable terms)
- How to **measure** the strength of a joint?
- What are possible **weaknesses** of a joint?

Specifications / Requirements

- ***Durability***
  - Thermal cycling [‘change of temperatures’ / ex. day/night, sunlight exposure, shadow, autoclave in medical environment, etc.]
  - Mechanical cycling [ex. connectors, cables, etc.]

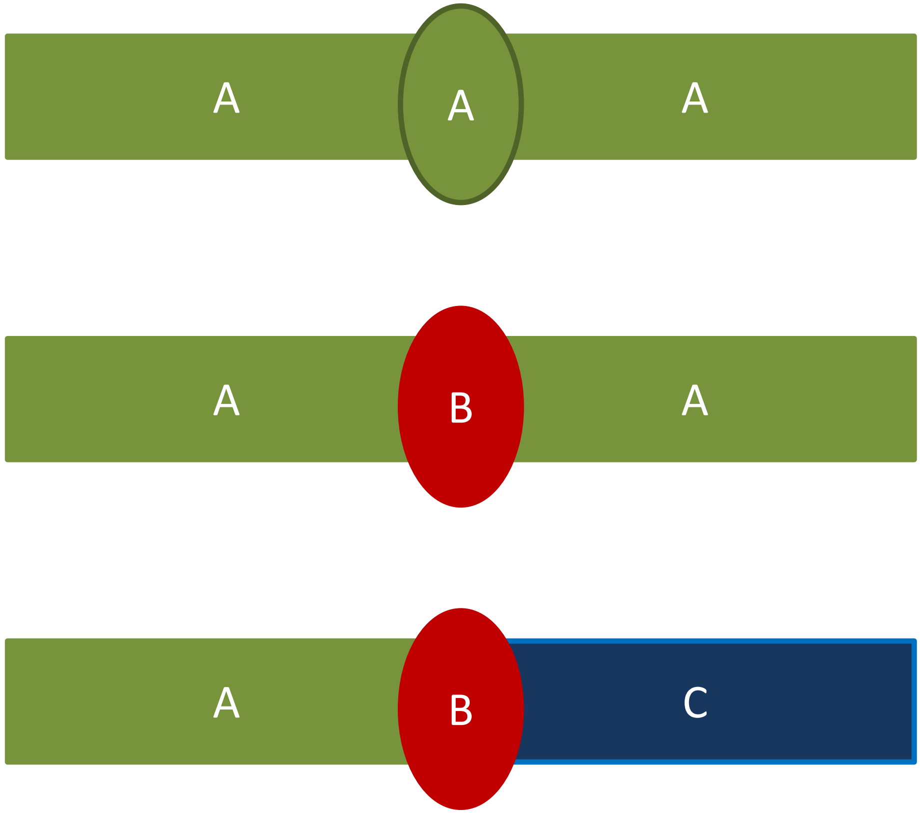
- ***Environmental***
  - Corrosion
  - Moisture (ex. humidity)
  - Light (ex. polymers)

Attaching two parts together...

- ‘Interfacing materials through melting’ Welding, brazing, soldering, etc.
- Gluing
- ‘Link through material contacting’:
  - Permanents, semi-permanents: rivets, force fit, optical contacting, etc.
  - Temporary: screws, clips, etc.

Welding

- Interface of the two parts to be welded are **molten**.



Homogeneous:
Two identical material, *with or without* added material

Heterogeneous:
Two identical material *with* a second material added

Heterogeneous: Three different materials

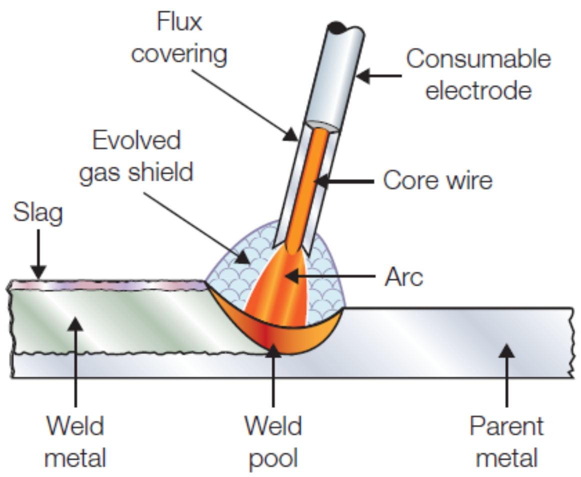
Soldering

- Examples of alloys used as solder materials (see follow-up lecture from V. Subramanian)

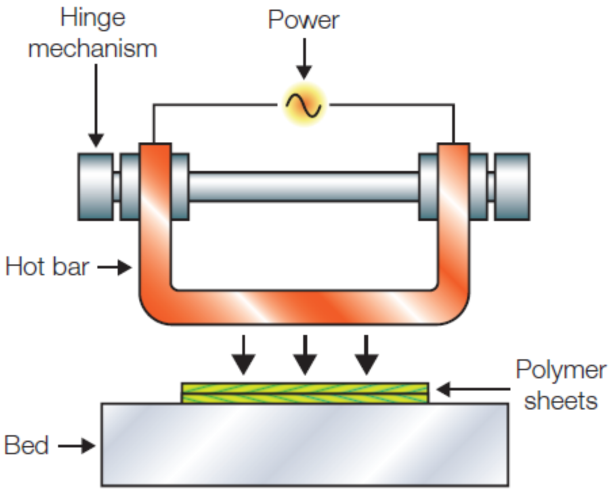
Solder	Typical application
Tin-lead	General purpose
Tin-zinc	Aluminum
Lead-silver	Strength at higher than room temperature
Cadmium-silver	Strength at high temperatures
Zinc-aluminum	Aluminum; corrosion resistance
Tin-silver-copper	Electronics
Tin-bismuth	Electronics

Source: Manufacturing Processes for Engineering Materials, Sixth Edition
Serope Kalpakjian | Steven Schmid

Welding



Metal arc welding

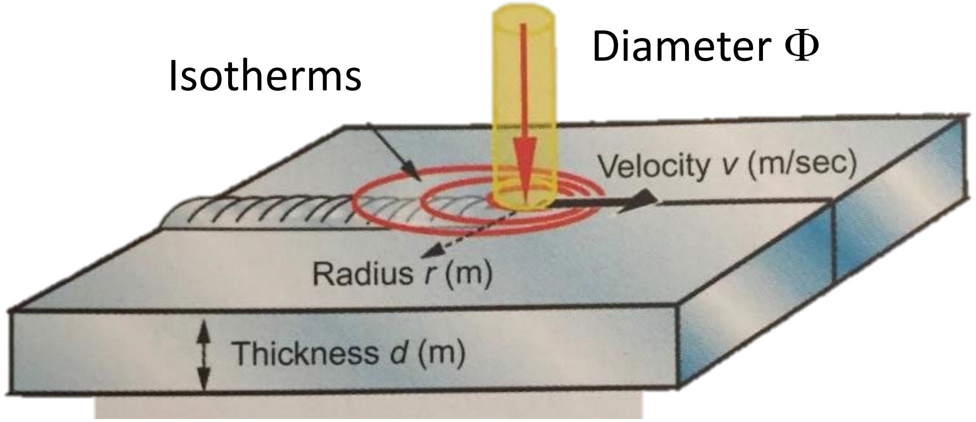


Hot bar polymer welding

(Illustrations: M. Ashby)

- **Arc welding**
- **Electron beam welding**
  - https://youtu.be/zX9tvdTEAEo
- **Friction welding**
  - https://youtu.be/dL9KTvVAEg8
  - https://youtu.be/8xXybQ_aCUM
- **Seam welding** (electrical current)
- **Laser welding**

Thermal field for a moving heat source



(**Thin plate** approximation, **d << Φ**)

$$T(t) = T_0 + \left(\frac{1}{\sqrt{4\pi k \rho C_p}}\right)\left(\frac{q}{vd}\right)\left(\frac{1}{\sqrt{t}}\right)e^{-\left(\frac{r^2}{4\lambda t}\right)}$$

Heat rate (J/s)

Lateral distance from the heat source

Diffusivity

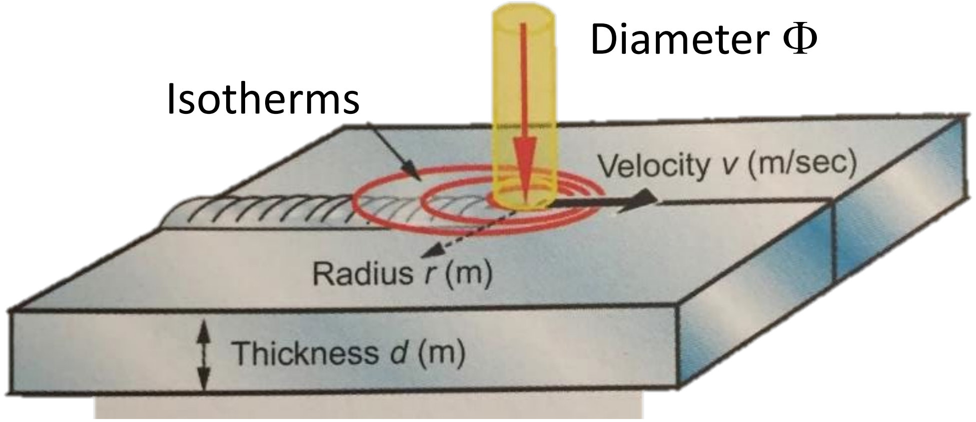
Plate thickness (m)

Scanning speed (m/s)

Material constants (**k:** conductivity, **ρ:** density, **Cp:** heat capacity)

$$T_{Max} = T_0 + \sqrt{\left(\frac{2}{\pi e}\right)}\left(\frac{1}{2\rho C_p}\right)\left(\frac{q}{vd}\right)\left(\frac{1}{r}\right)$$

Thermal field for a moving heat source



(**Thick plate** approximation, **d >> Φ**)

$$T(t) = T_0 + \left(\frac{1}{2\pi k}\right)\left(\frac{q}{v}\right)\left(\frac{1}{t}\right)e^{-\left(\frac{r^2}{4\lambda t}\right)}$$

Heat rate (J/s)

Lateral distance from the heat source

Diffusivity

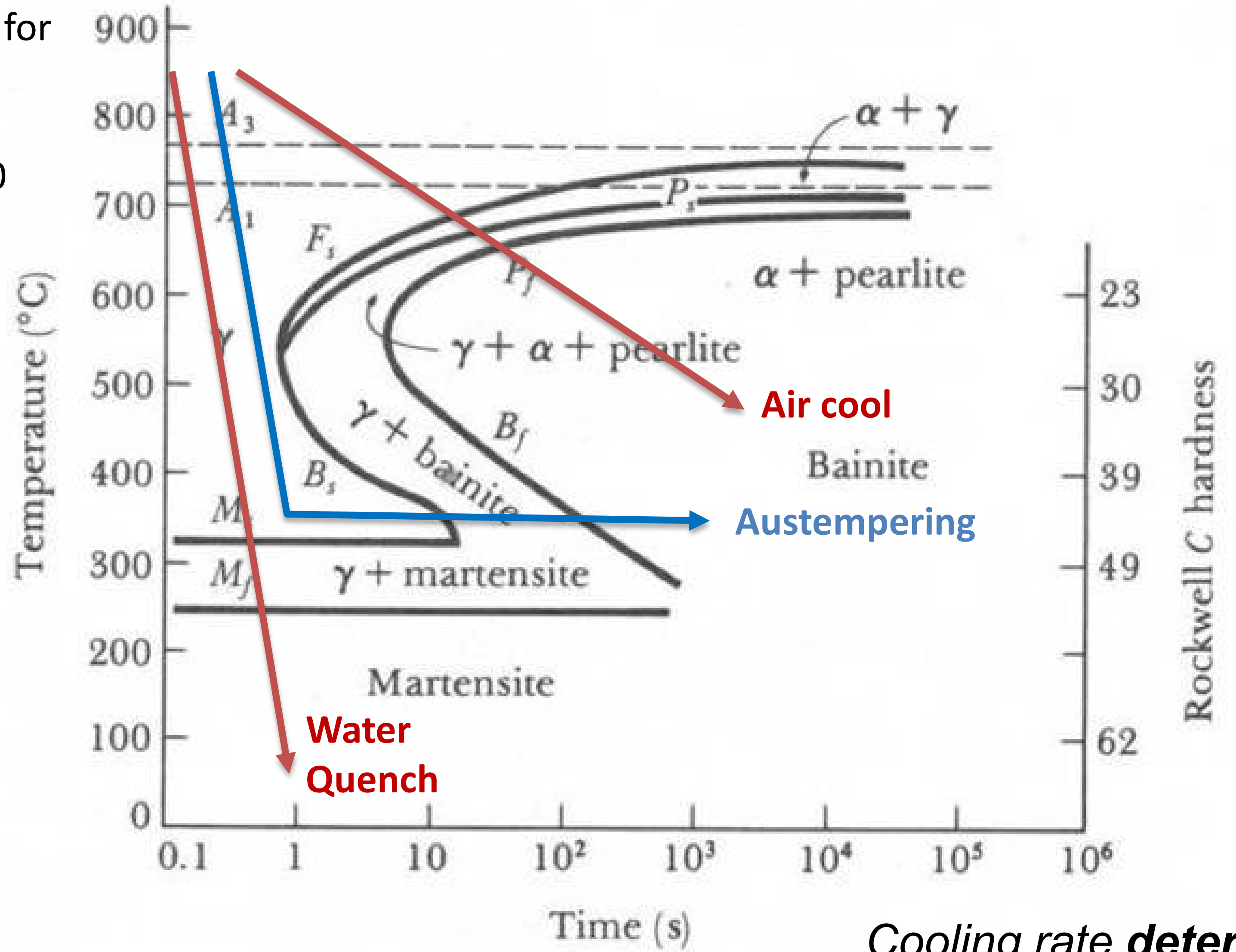
Material constants (k: conductivity)

Scanning speed (m/s)

$$T_{Max} = T_0 + \left(\frac{2}{\pi e}\right)\left(\frac{1}{\rho C_p r^2}\right)\left(\frac{q}{v}\right)$$

Why cooling rate matters?

‘TTT’ curve for Carbon Steel AISI 1050



*Cooling rate **determines** the microstructure…*

Brazing

- Unlike a welding, the part to be assembled **are not melted**
- Preserve the material part
- **Diffusion process**
- **Soldering** is a lower-temperature version of brazing.

First brazing…

- Gold dog, *Suse, Iran*

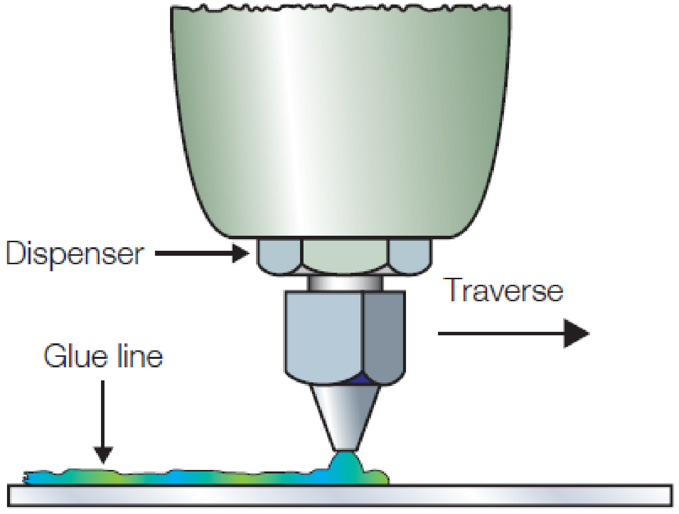
(-3100/-3300 BC)

(Photo: Musée du Louvre)

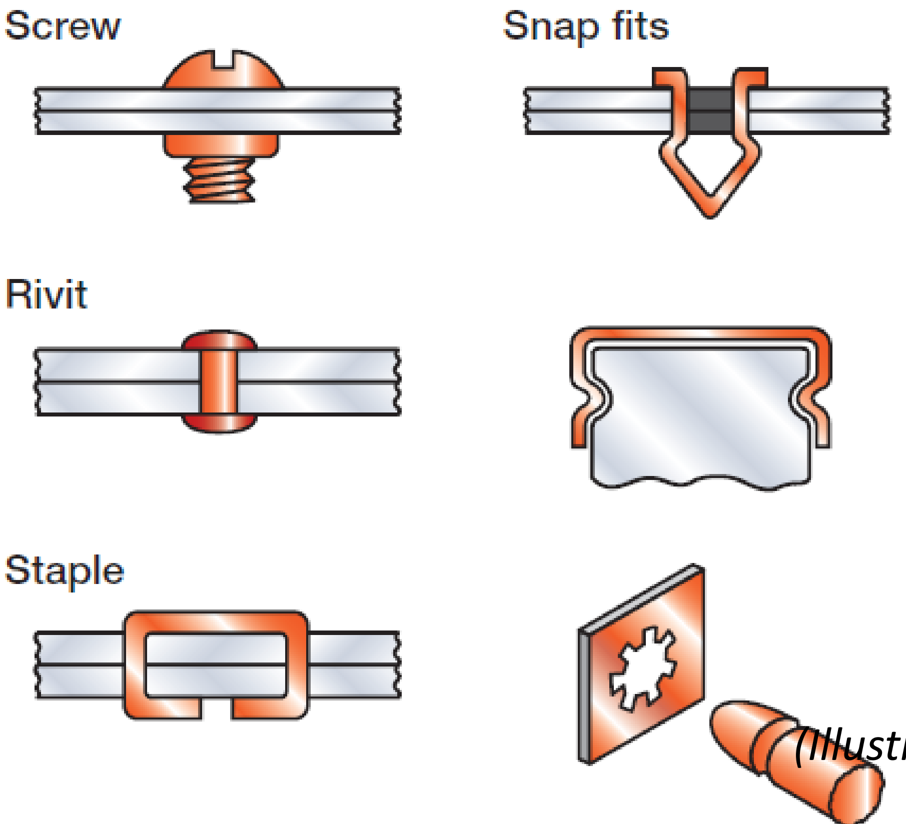
DUVAL Alain-René, ELUERE Christiane, HURTEL Loïc, TALLON Françoise, "La Pendeloque au chien de Suse. Étude en laboratoire d'une brasure antique", in *Revue du Louvre*, 1987, n 3, pp. 176-179.

Other attachment methods

- **Force fit** ('Chassage')
  - https://youtu.be/h-W9nkuxwgc
- **Gluing**
  - https://youtu.be/QEIQZ36V7Fg?list=PLiJi6vHjLILYQWBRaq-PPt1Or52slzpNv
- **Mechanical fasteners** (Rivet, staple, etc.)



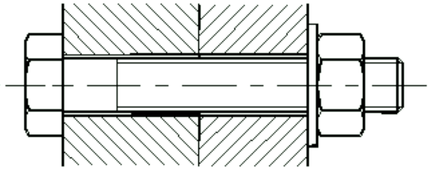
(gluing)

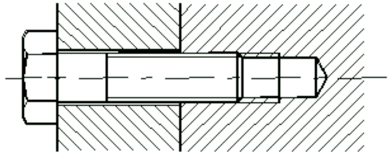


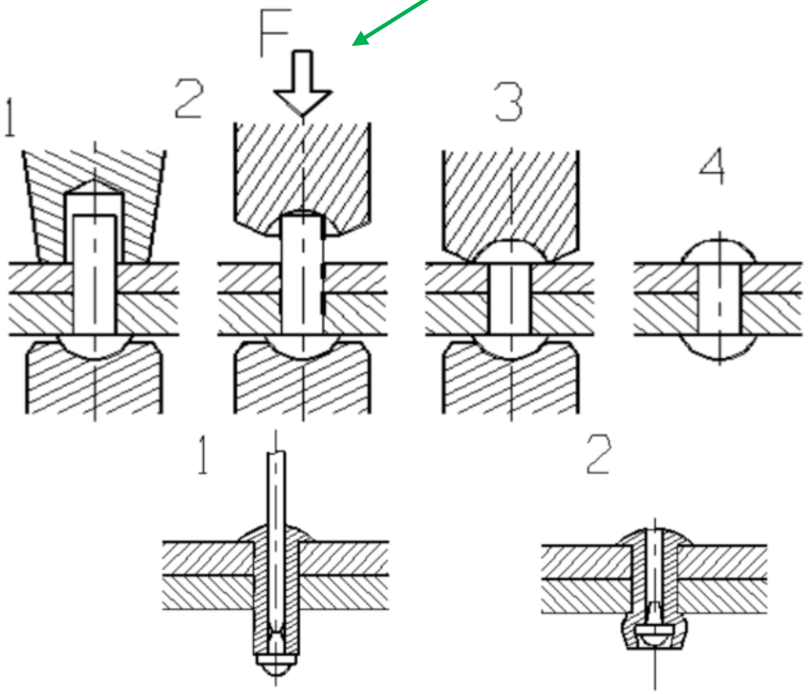
(Illustrations: M. Ashby)

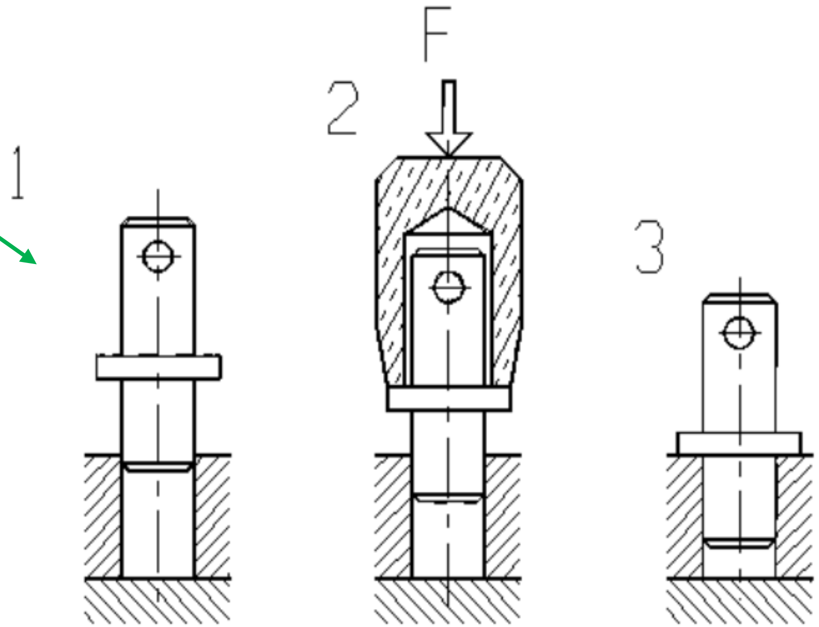
Classical mechanical assembly

- Nuts & Bolts
- Screws
- Force-fit
- Rivets







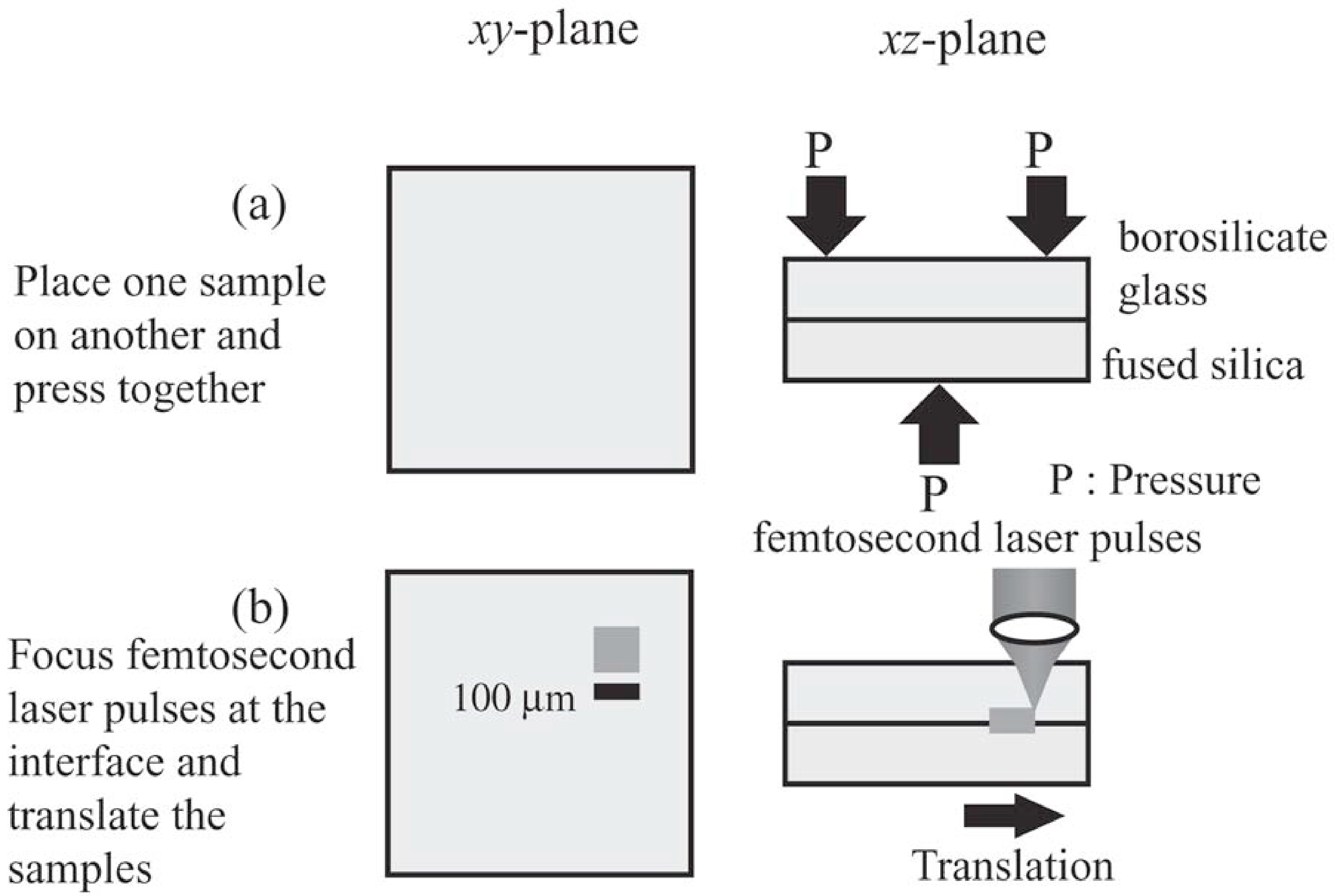


Less known…

- **Optical contacting (Swiss Optics)**
  - https://youtu.be/xZTdurPlhXE
- Glue-less methods
- No interfacial material
- Requires extremely low roughness (Ra ~ nm range) and flatness => Makes use of adhesion forces

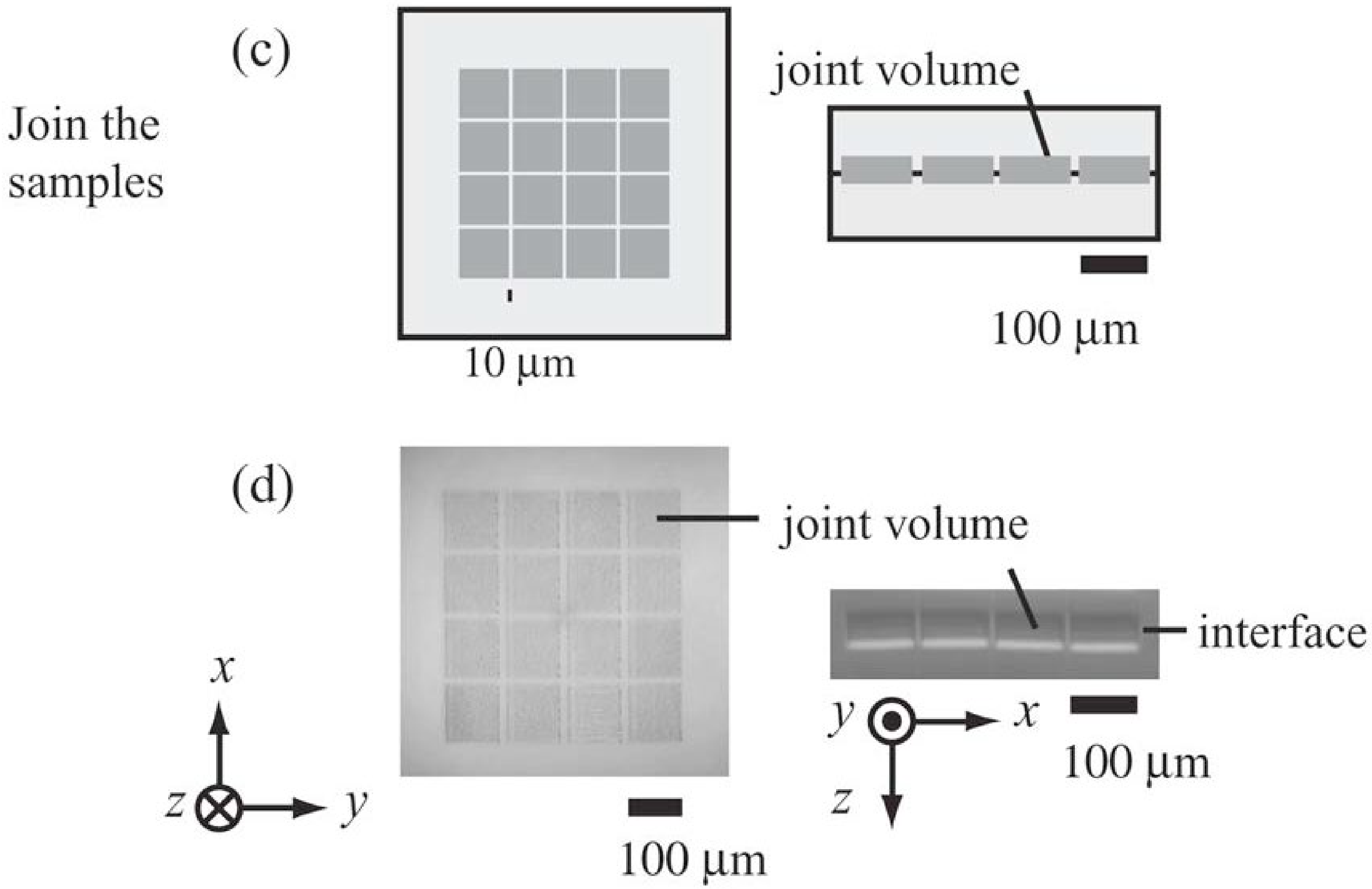
(source: 'Wizard of Vaz' / https://youtu.be/se3K_MWR488)

Non-linear laser welding (transparent materials)



W. Watanabe, et al., "Space-selective laser joining of dissimilar transparent materials using femtosecond laser pulses," Applied Physics Letters 89, 021106 (2006).

Non-linear laser welding (transparent materials)



Key things to remember

1. **Basic concept of DFM/A** – Design For Manufacturing / Design For Assembly
2. Generic construction principles for minimizing assembly errors
3. How to evaluate the cost of an assembly task
4. Overview of attachment/joining methods

Extra slides about mobility analysis
(the dual problem of assembly)
For exploring the topic deeper
(not considered in the exam)

Notion of assembly from the point of view of cancelling mobilities

- **Holding parts together**
- **Aligning** and **positioning** of elements one with respect to the other
- **Cancelling/controlling** degrees of freedoms
- **Maintaining** a position after assembly

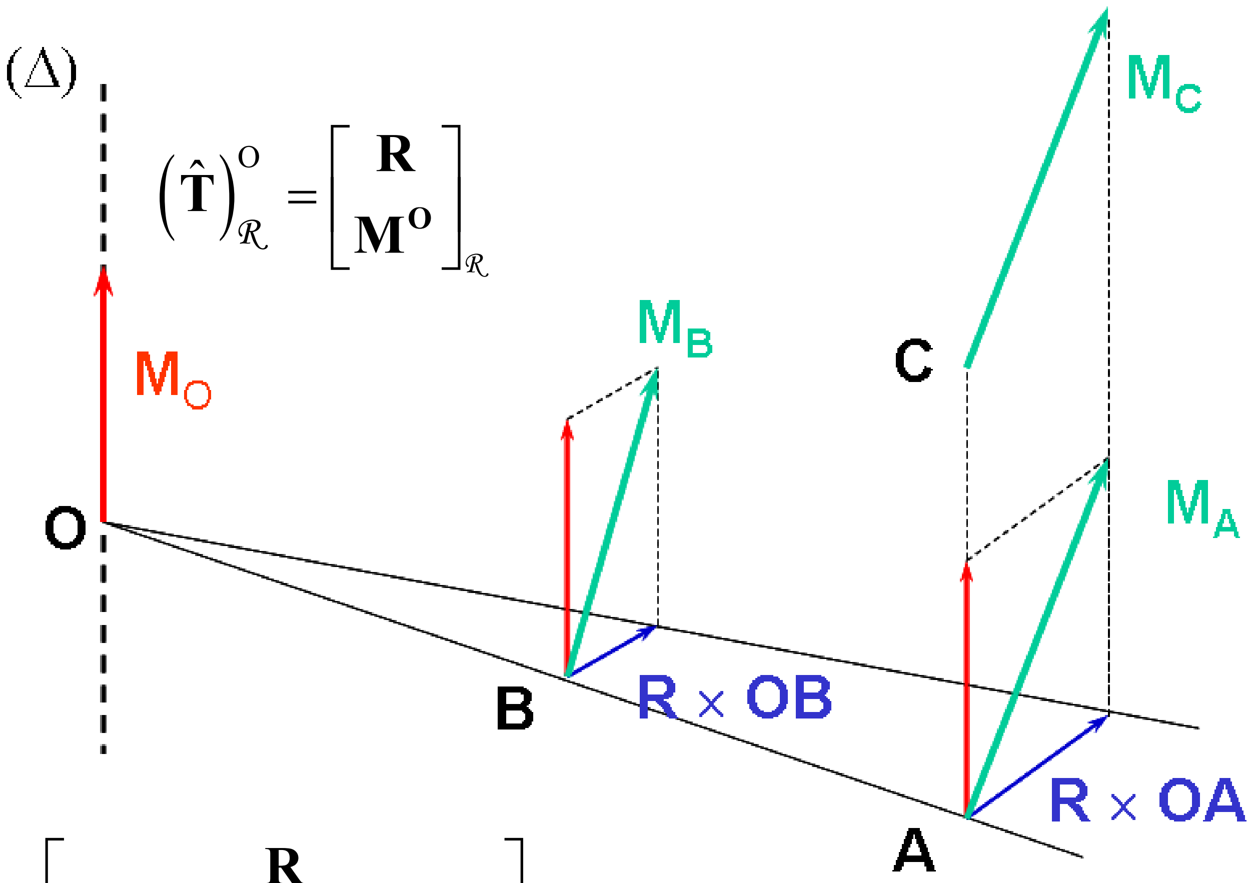
⇒

Mechanism theory, mobility and hyperstaticity analysis

Some tools for analyzing a mechanism and an assembly

- Theory of mechanisms
- **Mobility** and **constraints**
- **Duality** assembly / mechanism

Spatial vector



$$\left(\hat{\mathbf{T}}\right)_{\mathcal{R}}^{\mathrm{O}} = \begin{bmatrix} \mathbf{R} \\ \mathbf{M^O} \end{bmatrix}_{\mathcal{R}}$$

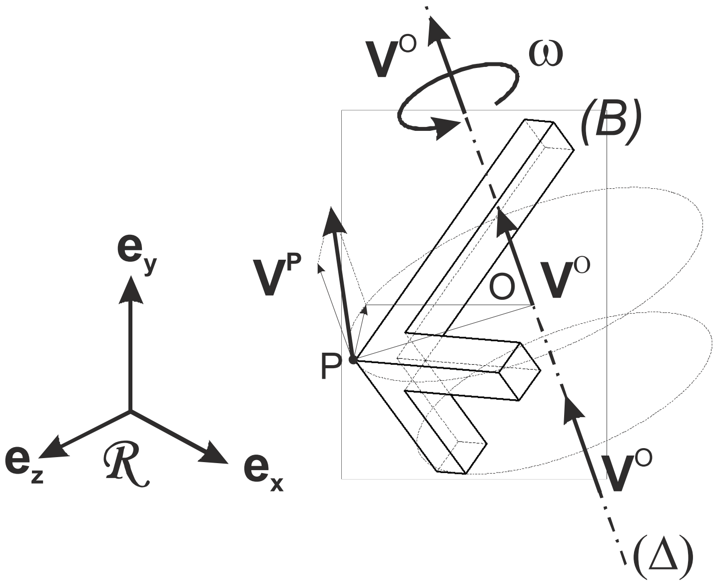
$$\left(\hat{\mathbf{T}}\right)_{\mathcal{R}}^{\mathrm{B}} = \begin{bmatrix} \mathbf{R} \\ \mathbf{M^A} = \mathbf{M^O} + \mathbf{R} \times \mathbf{OB} \end{bmatrix}_{\mathcal{R}}$$

$$\left(\hat{\mathbf{T}}\right)_{\mathcal{R}}^{\mathrm{A}} = \begin{bmatrix} \mathbf{R} \\ \mathbf{M^A} = \mathbf{M^O} + \mathbf{R} \times \mathbf{OA} \end{bmatrix}_{\mathcal{R}}$$

Spatial vector: velocity
(also called “Twist”) / Torseur cinématique

$$\left(\hat{\mathbf{V}}^{P}\right)_{\mathcal{R}} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{V}^{P} \end{bmatrix}_{\mathcal{R}} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{V}^{P} = \mathbf{V}^{O} + \boldsymbol{\omega} \times \mathbf{OP} \end{bmatrix}_{\mathcal{R}} \quad , \quad O \in (\Delta)$$

“Velocity of point P with respect to coordinate frame R”



Notes:

- $\left(\hat{\mathbf{V}}^{M}\right)_{\mathcal{R}} = \begin{Bmatrix} \omega_x & V_x^M \\ \omega_y & V_y^M \\ \omega_z & V_z^M \end{Bmatrix}_{\mathcal{R}}$ are called the Plucker coordinates

- $\left(\hat{\mathbf{V}}\right)_{\mathcal{R}}^{O'} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{V}^{O'} \end{bmatrix}_{\mathcal{R}} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{V}^{O'} = \mathbf{V}^{O} + \underbrace{\boldsymbol{\omega} \times \mathbf{OO'}}_{\mathbf{0}} \end{bmatrix}_{\mathcal{R}} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{V}^{O} \end{bmatrix}_{\mathcal{R}}$

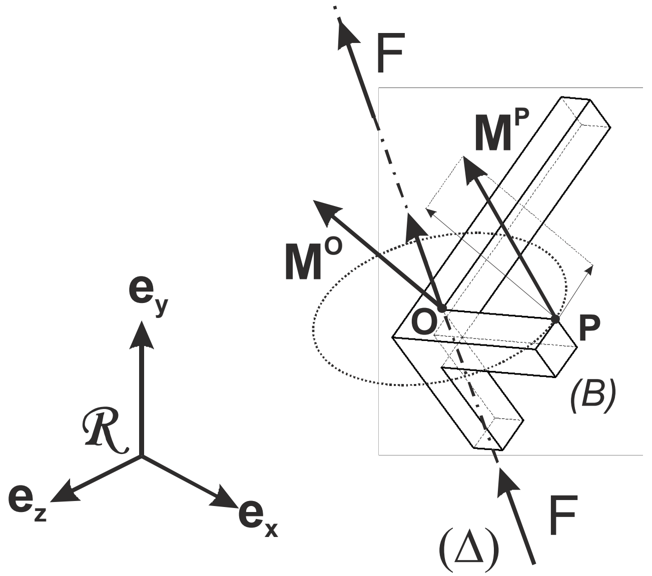
For any point O’ on the axis Δ

Spatial vector: **force**

(also called “Wrench” / torseur de force)

$$\left(\hat{\mathbf{F}}^{P}\right)_{\mathcal{R}} = \begin{bmatrix} \mathbf{F} \\ \mathbf{M}^{P} \end{bmatrix}_{\mathcal{R}} = \begin{bmatrix} \mathbf{F} \\ \mathbf{M}^{P} = \mathbf{M}^{O} + \mathbf{F} \times \mathbf{OP} \end{bmatrix}_{\mathcal{R}}$$

“Force applied by body n on body m”

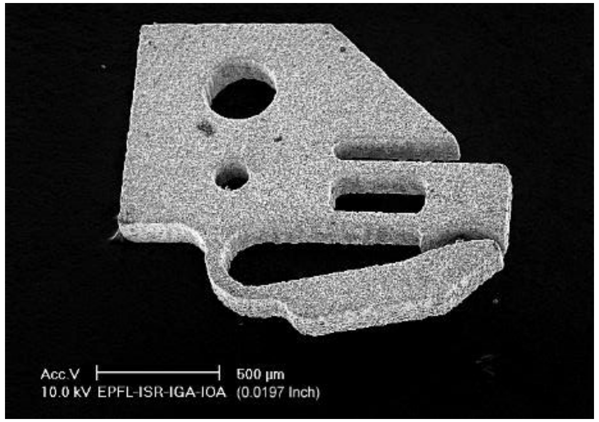


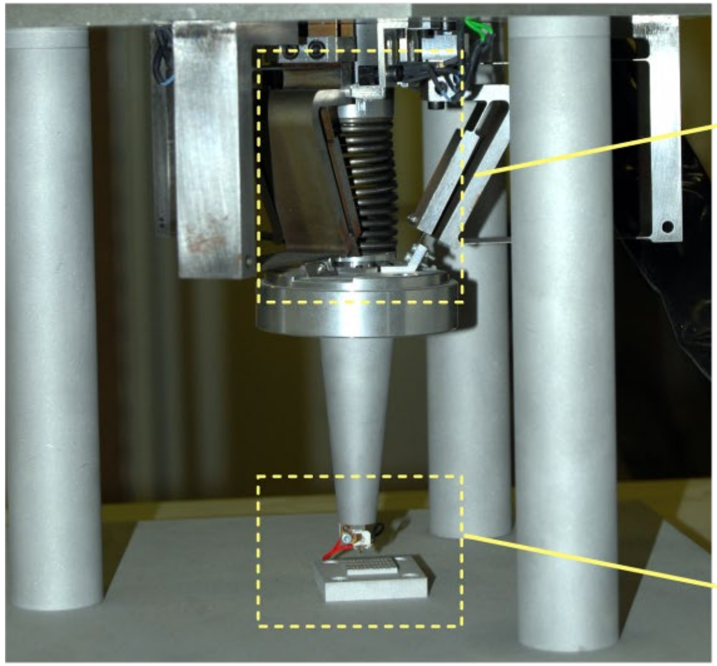
Notes:

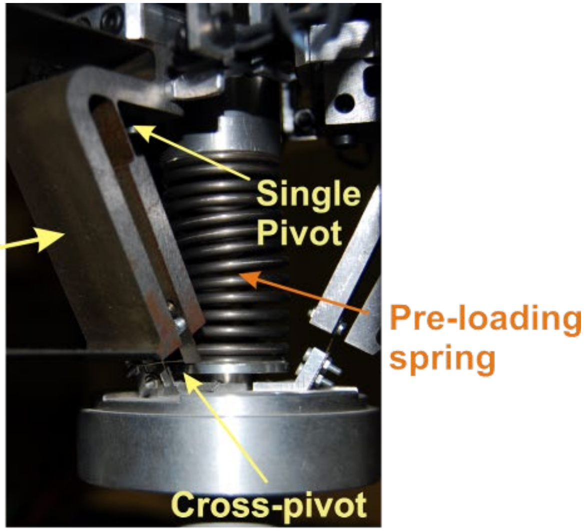
- $\left(\hat{\mathbf{F}}\right)^{M}_{\Re} = \begin{Bmatrix} F_x & M_x^M \\ F_y & M_y^M \\ F_z & M_z^M \end{Bmatrix}_{\Re}$ are the Plucker coordinates
- $\left(\hat{\mathbf{F}}\right)^{O'}_{\mathcal{R}} = \begin{bmatrix} \mathbf{F} \\ \mathbf{M}^{O'} \end{bmatrix}_{\mathcal{R}} = \begin{bmatrix} \mathbf{F} \\ \mathbf{M}^{O'} = \mathbf{M}^{O} \end{bmatrix}_{\mathcal{R}}$ if $O', O \in (\Delta)$

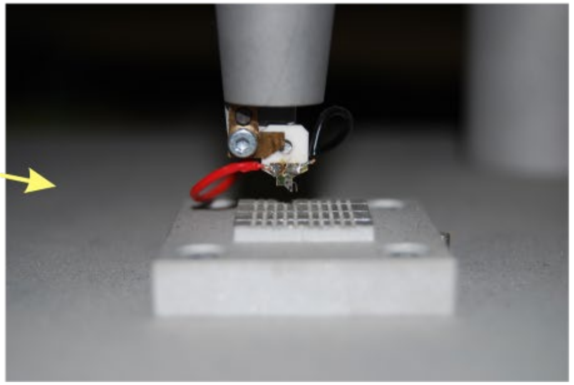
Illustration in microrobotics: Micro-robot platform with SMA gripper

- Three degree of motion
- Shape Memory Alloys gripper









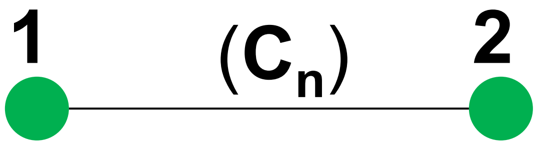
Goal of mobility analysis here is to be able to predict how the motion of the grippers can be correlated to the motion of the actuators that make the parallel robot moving. A second goal is to determine internal mobility that may exist in the system.

Graph theory: principle

- **Goal:** Mobility analysis
- **Graphical representation** of a mechanism
- Does not contain geometrical information!

Graph theory: Method

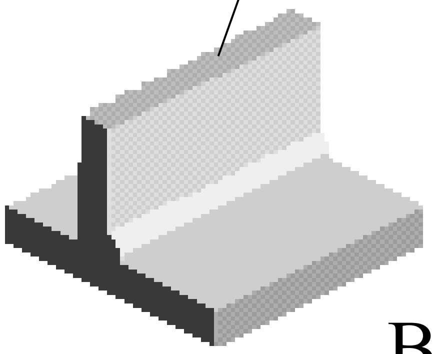
- One node => one body
- One joint between two bodies => one line connecting two nodes



(C_n) indicates the joint class
(n <=> number of degree-of-freedom)

C_0 / C_6 – Degree(s) of Freedom (trivial cases)

- Any unconstrained body has six degree-of-freedom. (C_6)

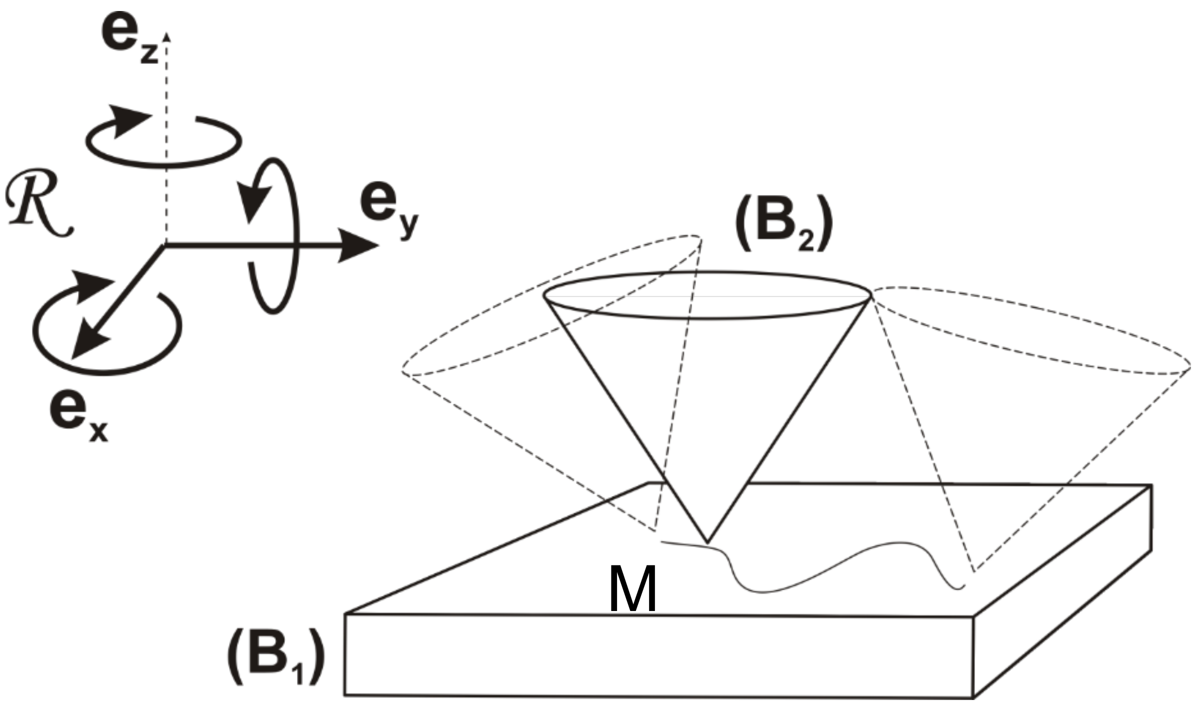


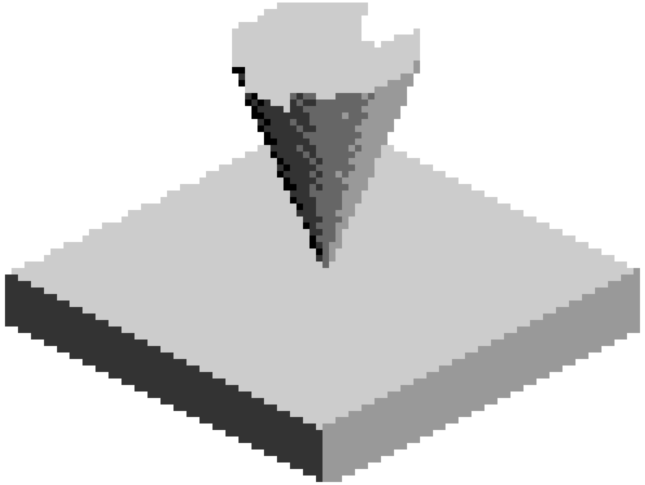
$$\left(\hat{\mathbf{F}}_{B_2 \to B_1}\right)^{M}_{\mathcal{R}} = \begin{Bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_{\mathcal{R}} \qquad \left(\hat{\mathbf{V}}\right)^{M}_{\mathcal{R}} = \begin{Bmatrix} \omega_x & V_x^M \\ \omega_y & V_y^M \\ \omega_z & V_z^M \end{Bmatrix}_{\mathcal{R}}$$

- Any fully constrained body has zero degree-of-freedom. (C_0)

$$\left(\hat{\mathbf{F}}_{B_2 \to B_1}\right)^{M}_{\mathcal{R}} = \begin{Bmatrix} F_x & M_x^M \\ F_y & M_y^M \\ F_z & M_z^M \end{Bmatrix}_{\mathcal{R}} \qquad \left(\hat{\mathbf{V}}\right)^{M}_{\mathcal{R}} = \begin{Bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_{\mathcal{R}}$$

C_5 – 5 Degrees of freedom joint

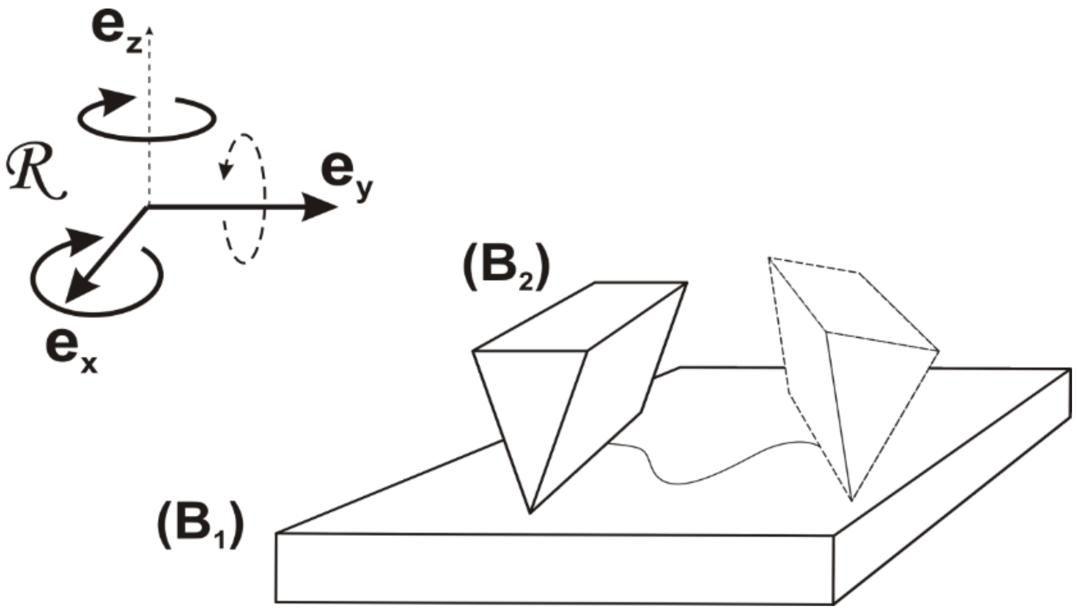


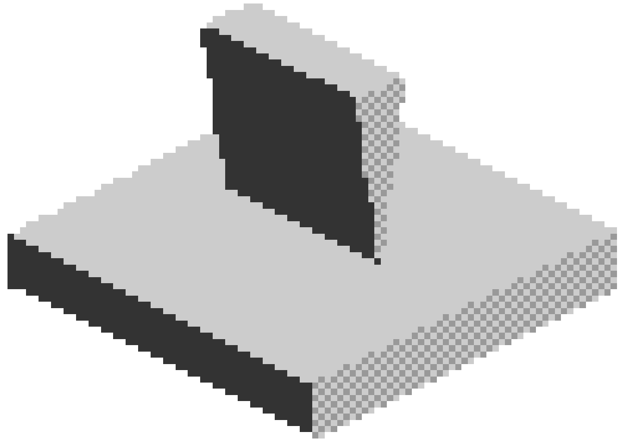


$$\left(\hat{\mathbf{F}}^{M}_{1\leftarrow 2}\right) = \begin{Bmatrix} 0 & 0 \\ 0 & 0 \\ F_z & 0 \end{Bmatrix}_{\mathcal{R}}$$

$$\left(\hat{\mathbf{V}}^{M}_{1/2}\right) = \begin{Bmatrix} \omega_x & V^M_x \\ \omega_y & V^M_y \\ \omega_z & 0 \end{Bmatrix}_{\mathcal{R}}$$

C_4 – 4 degrees of freedom joint

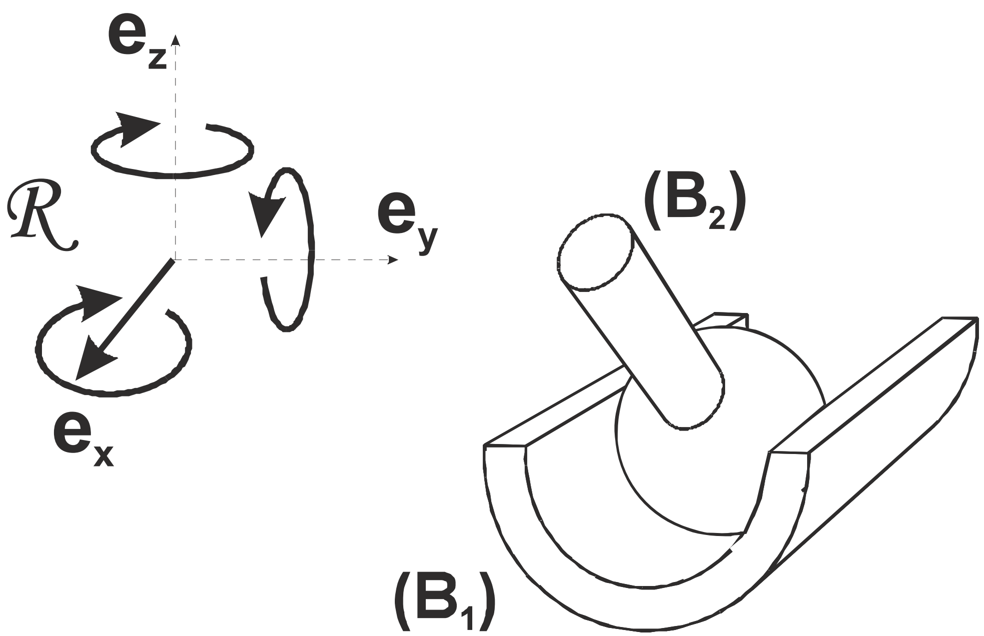


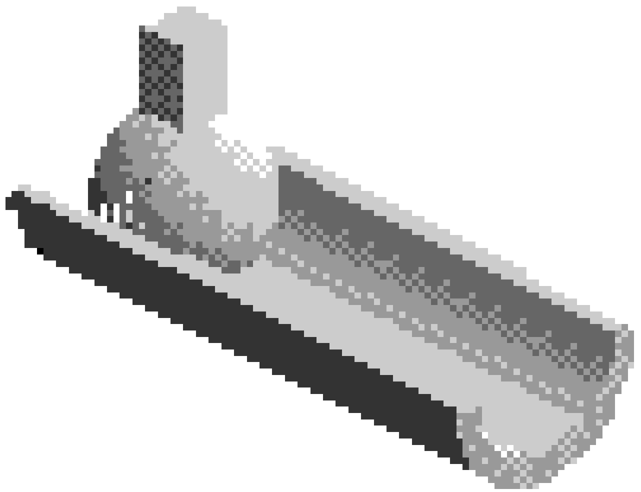


$$\left(\hat{\mathbf{F}}^{M}_{1\leftarrow 2}\right) = \begin{Bmatrix} 0 & 0 \\ 0 & M^{M}_{y} \\ F_z & 0 \end{Bmatrix}_{\mathcal{R}}$$

$$\left(\hat{\mathbf{V}}^{M}_{1/2}\right) = \begin{Bmatrix} \omega_x & V^{M}_{x} \\ 0 & V^{M}_{y} \\ \omega_z & 0 \end{Bmatrix}_{\mathcal{R}}$$

C_4 – 4 degrees of freedom joint

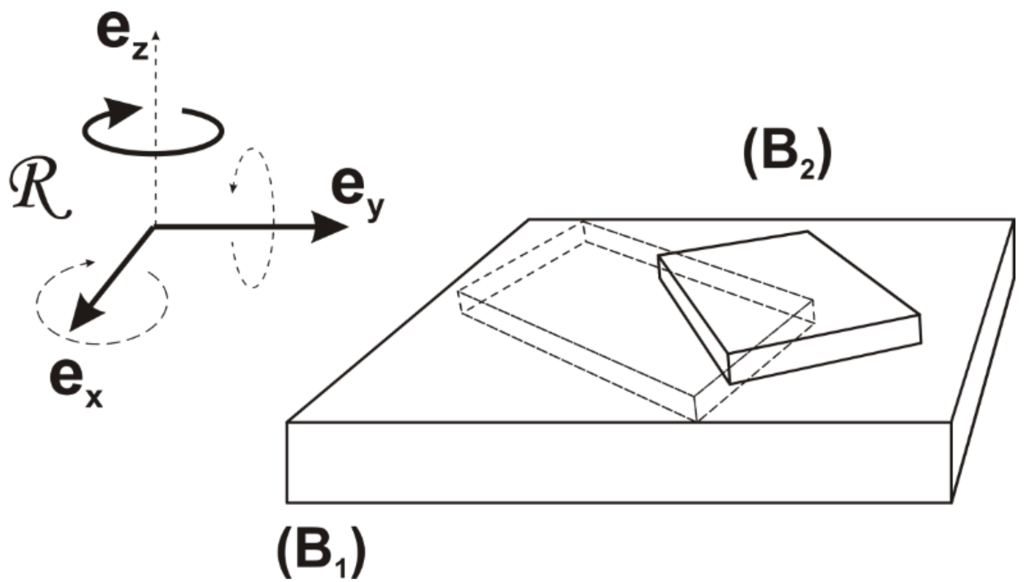


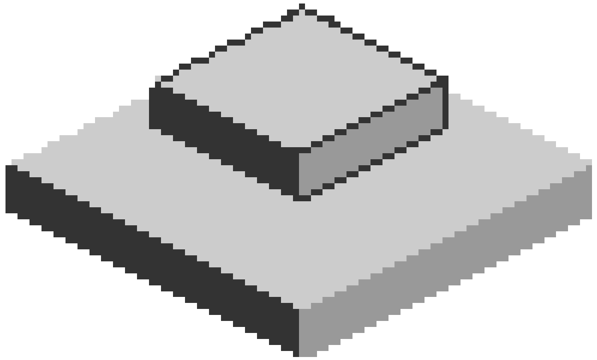


$$\left(\hat{\mathbf{F}}^{M}_{1\leftarrow 2}\right) = \begin{Bmatrix} 0 & 0 \\ F_y & 0 \\ F_z & 0 \end{Bmatrix}_{\mathcal{R}}$$

$$\left(\hat{\mathbf{V}}^{M}_{1/2}\right) = \begin{Bmatrix} \omega_x & V^{M}_{x} \\ \omega_y & 0 \\ \omega_z & 0 \end{Bmatrix}_{\mathcal{R}}$$

C_3 – 3 degrees of freedom joint

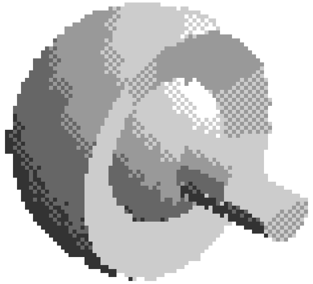


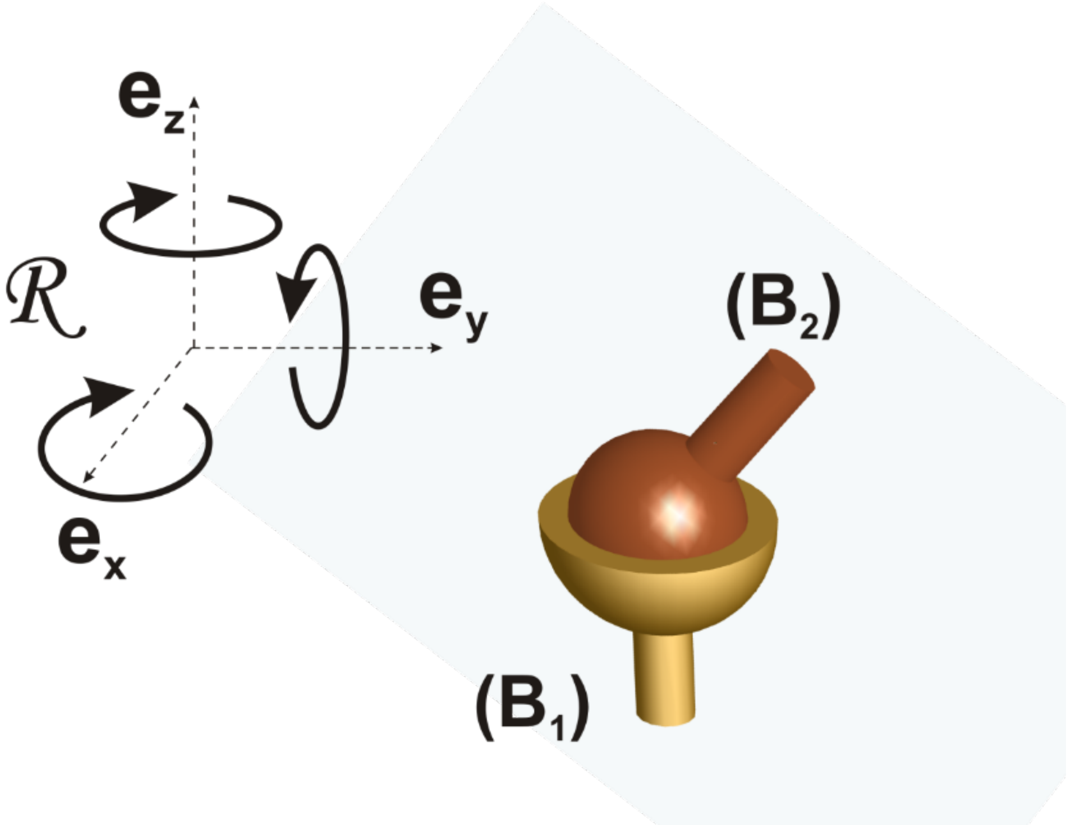


$$\left(\hat{\mathbf{F}}_{1\leftarrow 2}^{M}\right) = \begin{Bmatrix} 0 & M_x^M \\ 0 & M_y^M \\ F_z & 0 \end{Bmatrix}_{\mathcal{R}}$$

$$\left(\hat{\mathbf{V}}_{1/2}^{M}\right) = \begin{Bmatrix} 0 & V_x^M \\ 0 & V_y^M \\ \omega_z & 0 \end{Bmatrix}_{\mathcal{R}}$$

C_3 – 3 degrees of freedom

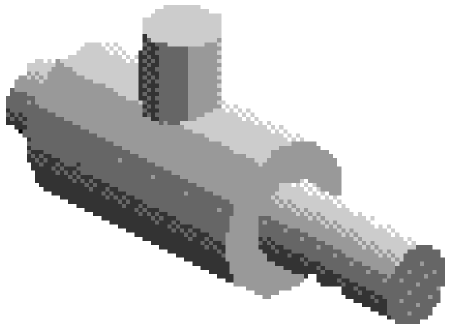


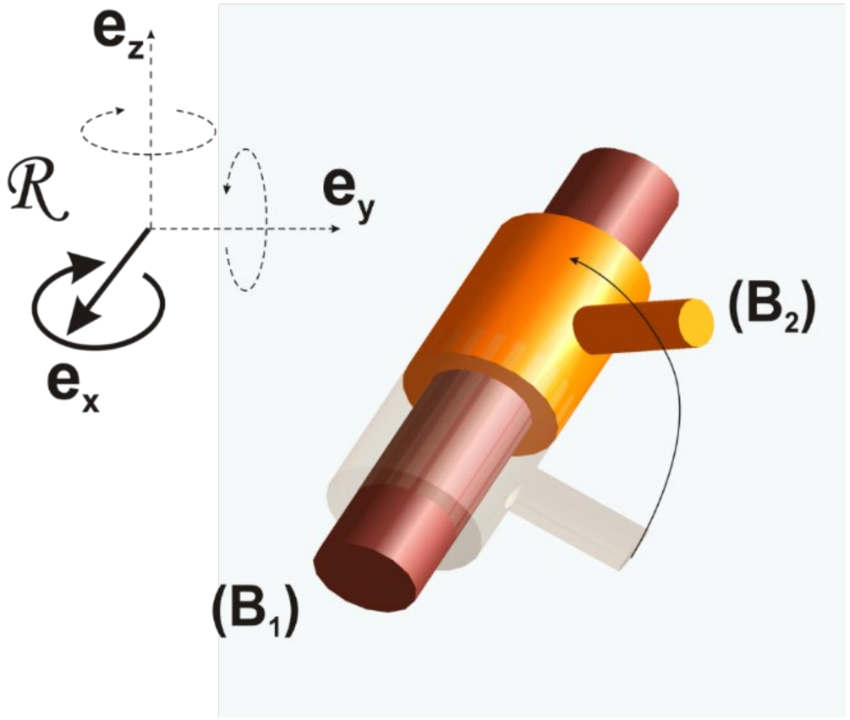


$$\left(\hat{\mathbf{F}}^{M}_{1\leftarrow 2}\right) = \begin{Bmatrix} F_x & 0 \\ F_y & 0 \\ F_z & 0 \end{Bmatrix}_{\mathcal{R}}$$

$$\left(\hat{\mathbf{V}}^{M}_{1/2}\right) = \begin{Bmatrix} \omega_x & 0 \\ \omega_y & 0 \\ \omega_z & 0 \end{Bmatrix}_{\mathcal{R}}$$

C_2 – 2 degrees of freedom joints

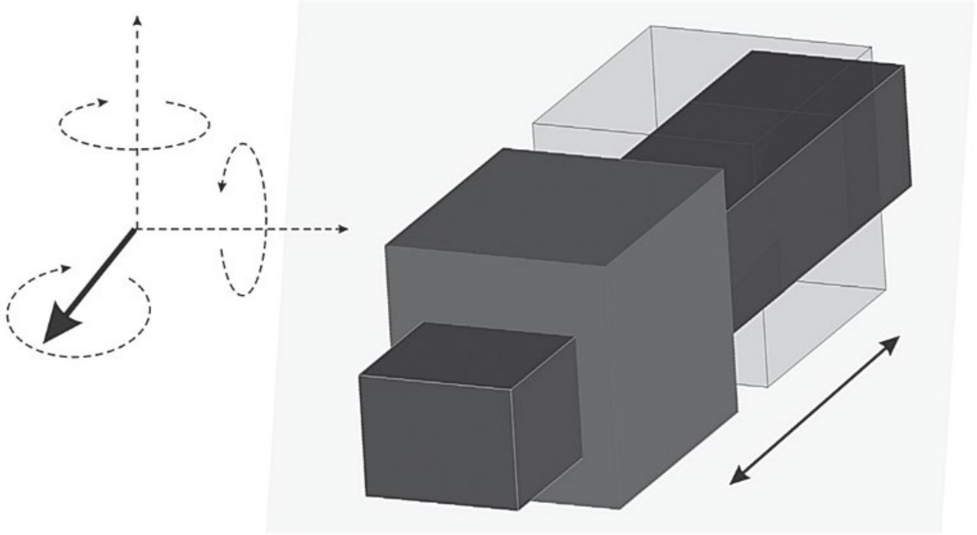




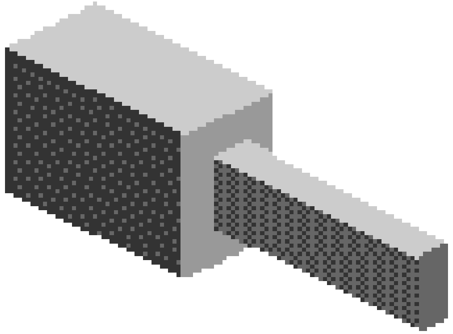
$$\left(\hat{\mathbf{F}}^{M}_{1\leftarrow 2}\right) = \begin{Bmatrix} 0 & 0 \\ F_y & M_y^M \\ F_z & M_z^M \end{Bmatrix}_{\mathcal{R}}$$

$$\left(\hat{\mathbf{V}}^{M}_{1/2}\right) = \begin{Bmatrix} \omega_x & V_x^M \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_{\mathcal{R}}$$

C_1 – 1 degree of freedom

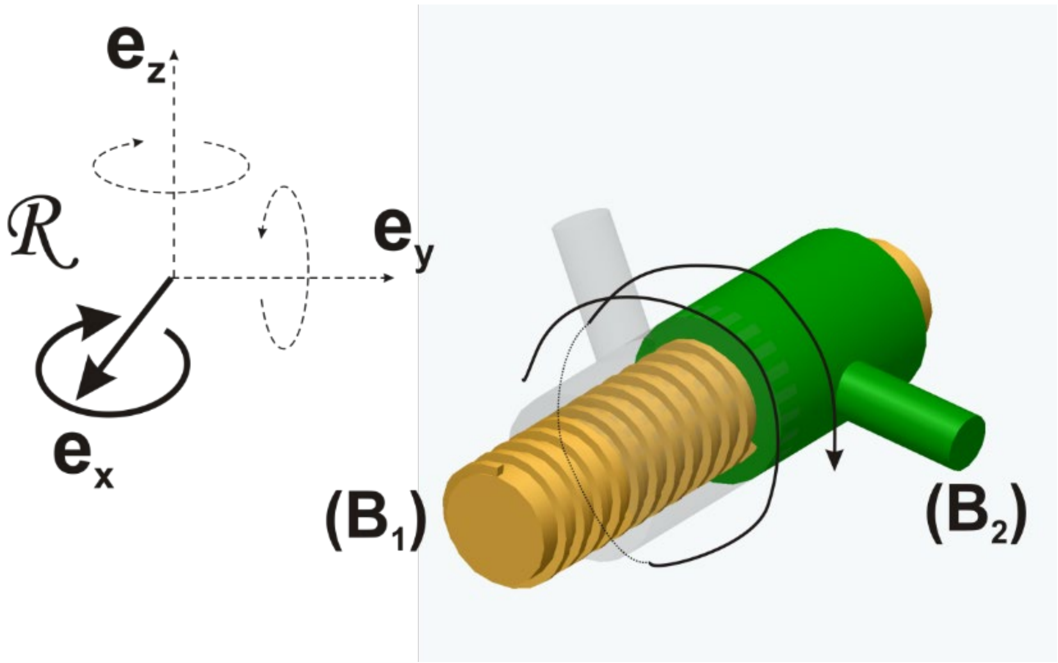


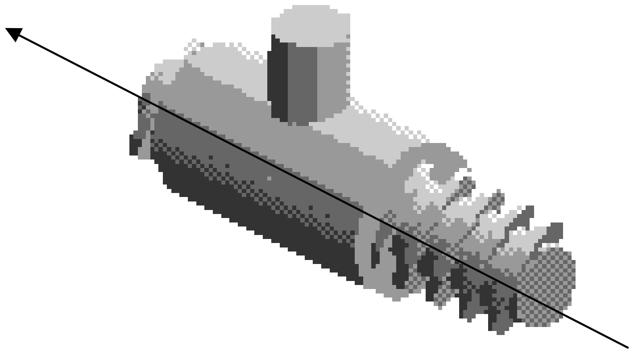
$$\left(\hat{\mathbf{F}}^{M}_{1\leftarrow 2}\right) = \begin{Bmatrix} 0 & M^{M}_{x} \\ F_y & M^{M}_{y} \\ F_z & M^{M}_{z} \end{Bmatrix}_{\mathcal{R}}$$



$$\left(\hat{\mathbf{v}}^{M}_{1/2}\right) = \begin{Bmatrix} 0 & V^{M}_{x} \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_{\mathcal{R}}$$

C_1 – 1 degree of freedom



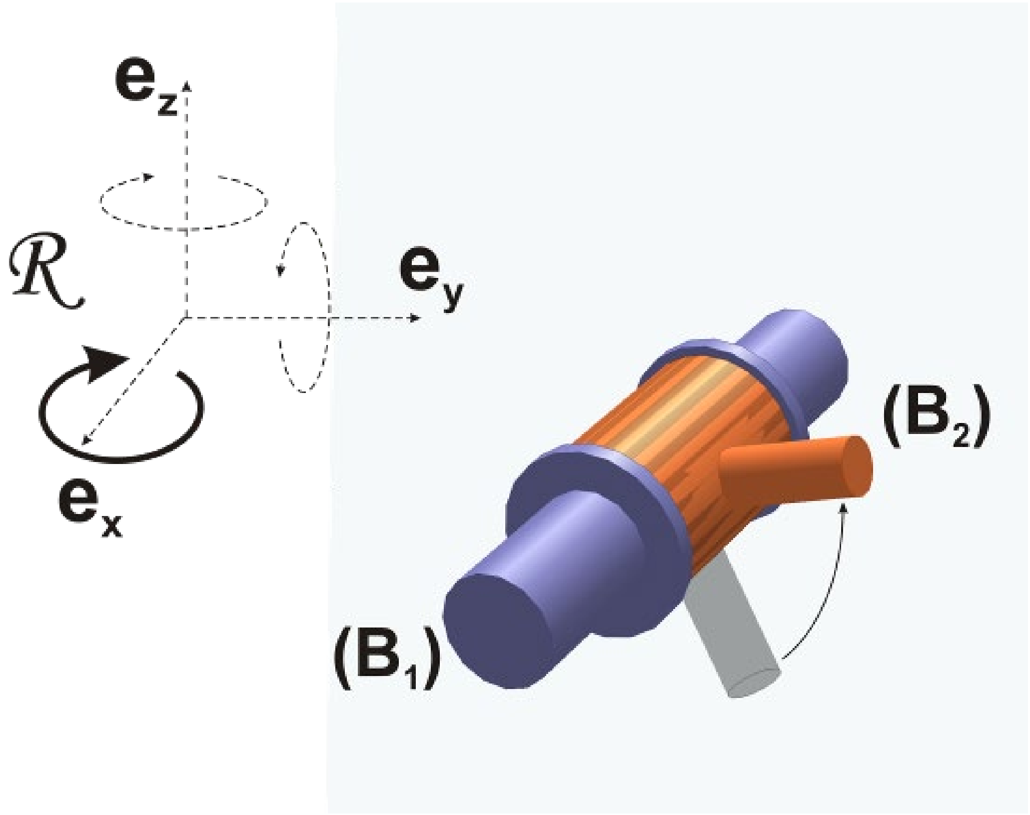


$$\left(\hat{\mathbf{F}}^{M}_{1\leftarrow 2}\right) = \begin{Bmatrix} 0 & 0 \\ F_y & M^M_y \\ F_z & M^M_z \end{Bmatrix}_{\mathcal{R}}$$

$$\left(\hat{\mathbf{V}}^{M}_{1/2}\right) = \begin{Bmatrix} \omega_x & V^M_x \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_{\mathcal{R}}$$

+ one relation: $V_x{}^M = \lambda\omega_x$

C_1 – 1 degree of freedom



$$\left(\hat{F}^{M}_{1\leftarrow 2}\right) = \begin{Bmatrix} 0 & M^{M}_{x} \\ F_{y} & M^{M}_{y} \\ F_{z} & M^{M}_{z} \end{Bmatrix}_{\mathcal{R}}$$

$$\left(\hat{v}^{M}_{1/2}\right) = \begin{Bmatrix} 0 & V^{M}_{x} \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_{\mathcal{R}}$$

Graph with close loops

From the Graph Theory:

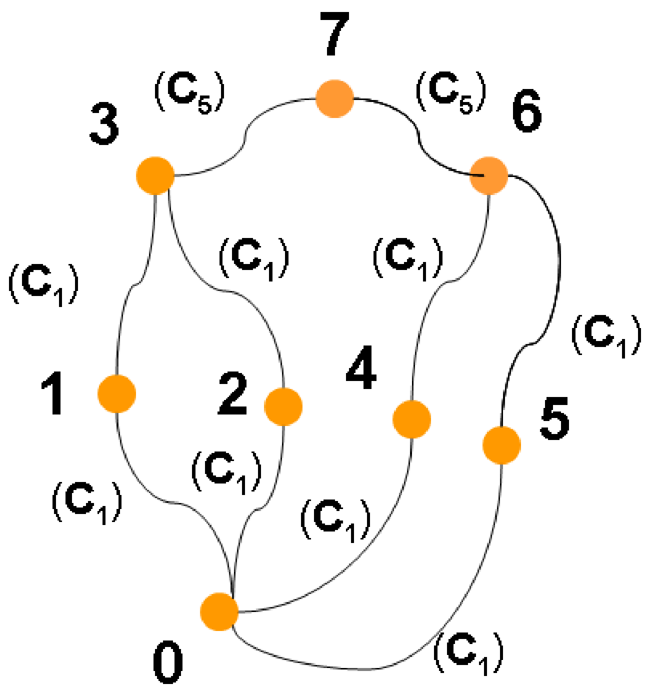
Let $\mathbf{N_L}$ the number of links and $\mathbf{N_b}$ the number of nodes

The number of independent loop n writes:

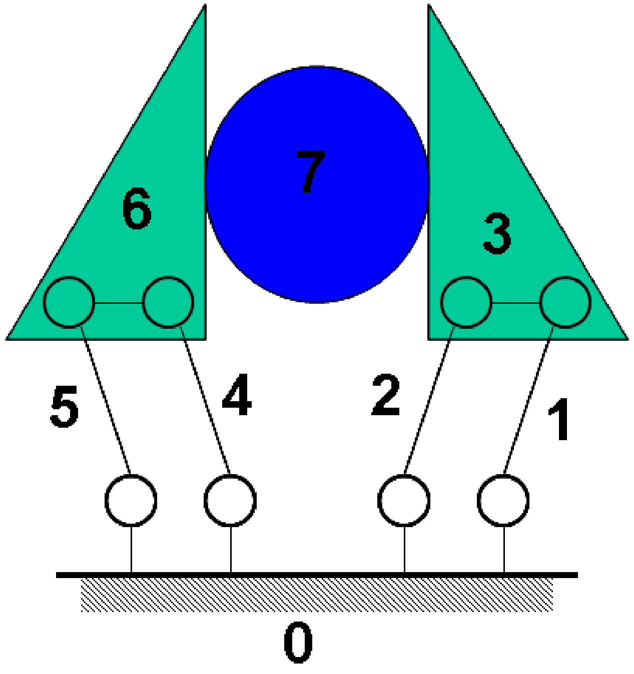
$$n = N_L - N_b + 1$$

Graph with close loops: illustration

Gripper mechanism



Graph of the mechanism



'Functional diagram'

n = 10 (links) - 8 (bodies) + 1 = 3 independent loops

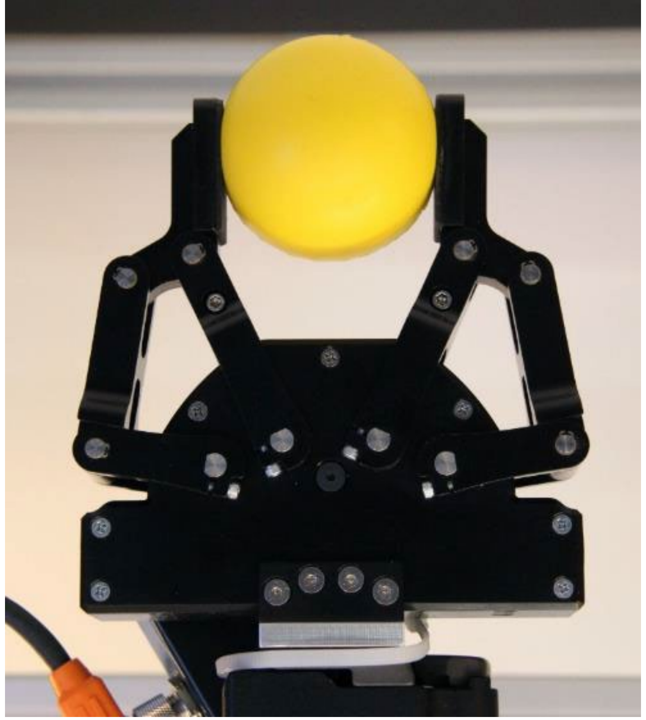


Illustration: M. Bélanger-Barrette (https://blog.robotiq.com)
Illustrative video to watch: https://youtu.be/DW4ttbLh_kA

Illustration: Makeblock (r)

Quiz: Can you spot the difference between the two?

Graph: analysis (I)

∃ n = N_L-N_B+1 independent loops

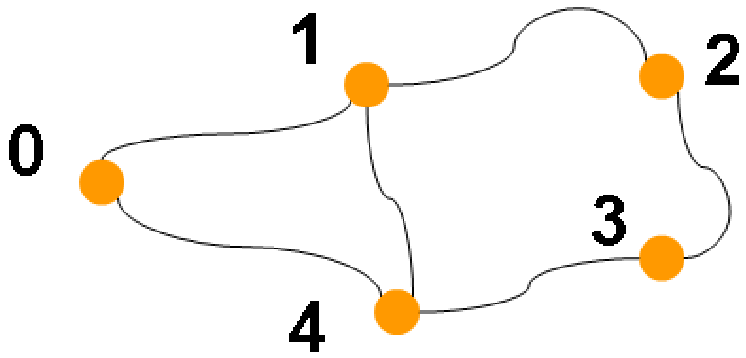
↓

n kinematics relations

↓

∃ E_k = 6.n independent kinematics scalar equations (in the space) / (3.n in the plane)

Example



$$n = 6 - 5 + 1 = 2$$

➡ 12 kinematics relations

For a loop (Chasles theorem): :

$$(\hat{v})^M_{0/0} = (\hat{v})^M_{0/1} + (\hat{v})^M_{1/4} + (\hat{v})^M_{4/0} = \hat{0}$$

➡ 6 equations

$$(\hat{v})^M_{0/0} = (\hat{v})^M_{0/1} + (\hat{v})^M_{1/2} + (\hat{v})^M_{2/3} + (\hat{v})^M_{3/4} + (\hat{v})^M_{4/0} = \hat{0}$$

➡ 6 equations

We could also choose the path 1 – 2 – 3 – 4 – 1

Mobility analysis: Kinematics approach (1st method)

- $\mathbf{U_k = E_k}$, Fully determined, as many unkown than equations, **m = 0**
- $\mathbf{U_k > E_k}$, Several possible solutions, **m > 0 /** $m = U_k – E_k$
- $\mathbf{U_k < E_k}$ => ? , More equations than unknown, over-constraint, **m < 0**

Kinematics / Kinetics unknown

- **Number of kinematics** unknown

$$U_k = \sum_{i=1}^{5} k_i C_i$$

- **Number of kinetics** unknown

$$U_f + U_k = 6 \cdot N_L \Rightarrow U_f = 6 \cdot N_L - \sum_{i=1}^{5} k_i C_i$$

Mobility point of view: Kinematics point of view

dim=U_k

dim=E_k

$$\underbrace{\begin{pmatrix} a_{00} & & \\ & \cdot & \\ & \cdot & a_{ij} \\ & & \end{pmatrix}}_{\text{Rank }[E_k]} \begin{pmatrix} U_{k_1} \\ \cdot \\ \cdot \\ \cdot \\ U_{k_n} \end{pmatrix} = \begin{pmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{pmatrix}$$

$$m = U_k - \mathrm{rank}[E_k]$$

Assembly point of view: Kinetics equivalent (dual of mobility)

$$
\text{dim}=E_f \left\{ \overbrace{\begin{pmatrix} a_{00} & & & & \\ & \text{Rank }[E_f] & & & \\ & & . & a_{ij} & \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}}^{\text{dim}=U_f} \right. \begin{pmatrix} U_{f_1} \\ . \\ . \\ . \\ U_{f_n} \end{pmatrix} = \begin{pmatrix} F_{ext_1} \\ . \\ . \\ . \\ F_{ext_n} \end{pmatrix}
$$

$$h = E_f - \text{rank}[E_k] \qquad\qquad m - h = U_k - E_f$$

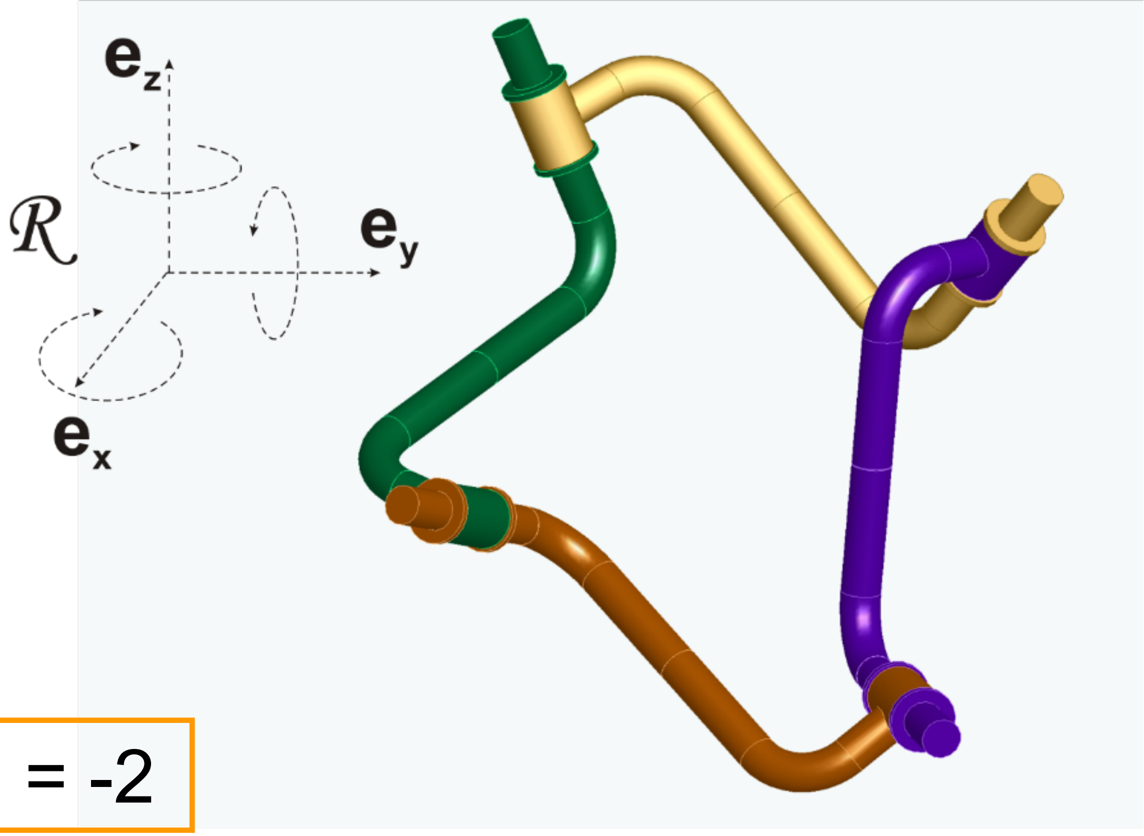
h is called the degree of **hyperstaticity**

2nd method: Chebyshev-Grübler-Kutzbach (CGK) Equation

$$\mathcal{M} = \sum_{i=1}^{5} n_i \cdot C_i - \underbrace{6 \cdot \left(N_L - N_B + 1\right)}_{\text{Graph}}$$

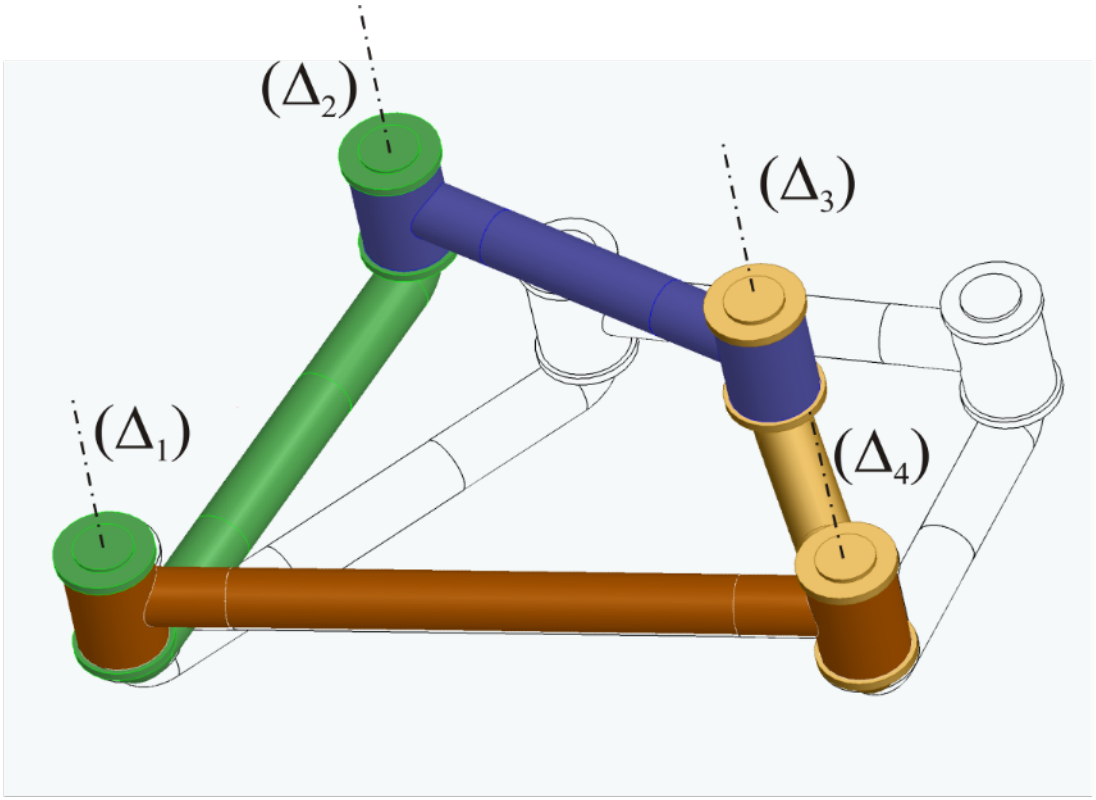
- Previous method is cumbersome
- Provide a faster way to calculate the mechanism mobility
- ***But…***

Example: Four bars mechanism (I)



m = -2

Example: Four bars mechanism (II)

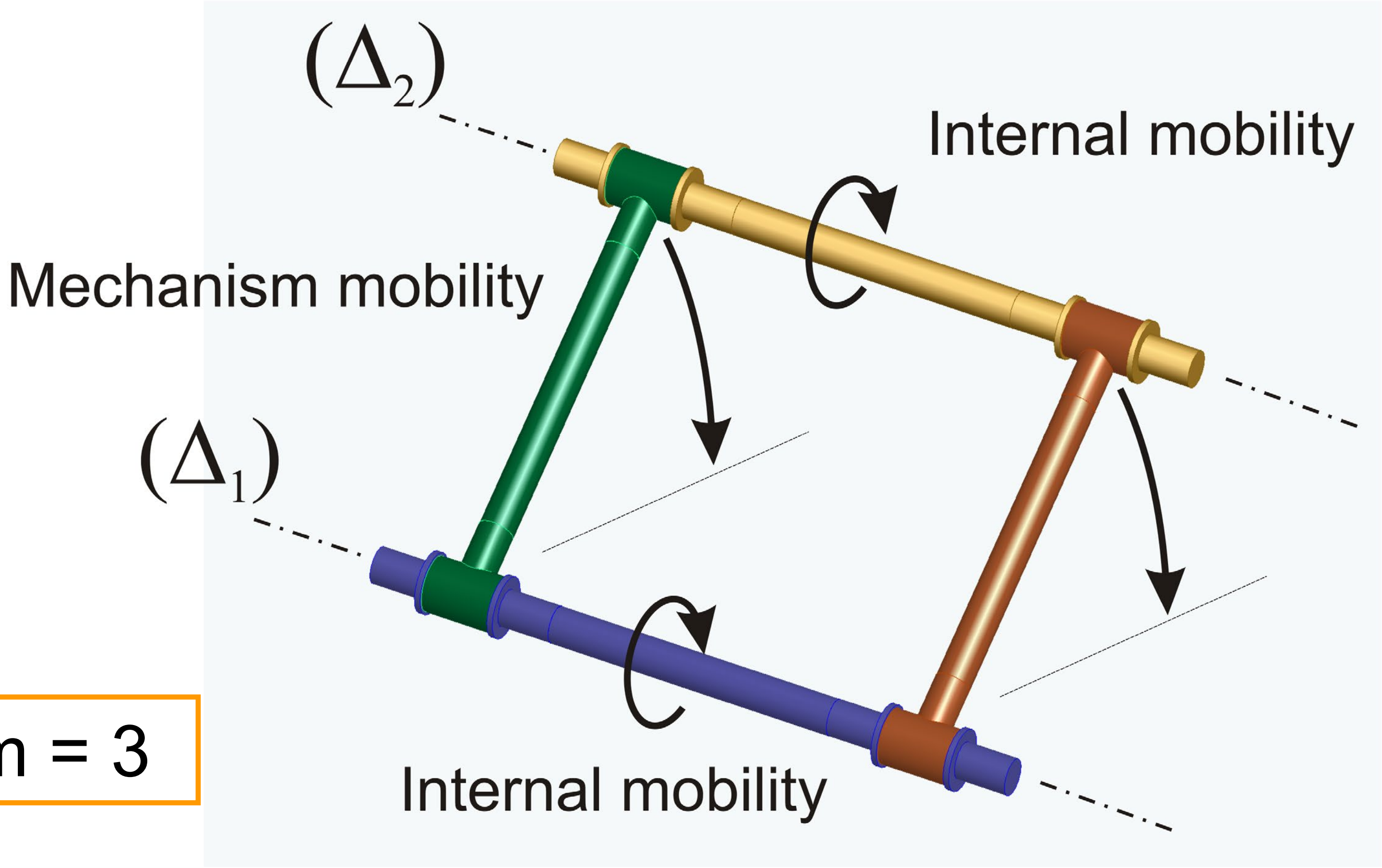


CGK equation give the wrong result !

Why?

m = -2 ➔ m = 1

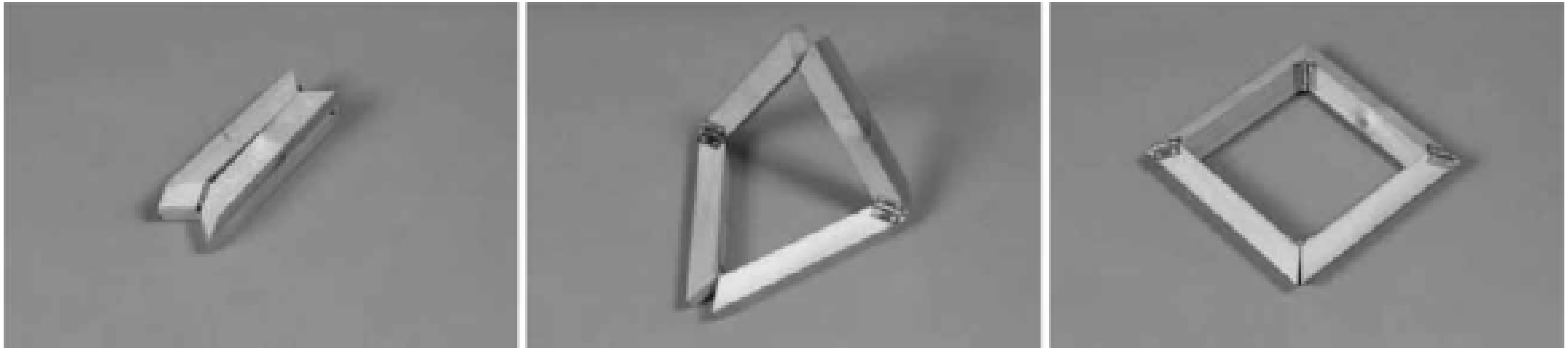
Example: Four bars mechanism (III)



m = 3

Note: this mechanism is redundant (a bar could be removed).

Example: Four bars mechanism (III)



Example of three-dimensional mechanism with four bars, and yet with **one degree of freedom**. (Bennett mechanism.) The axis orientations of the joints form two pairs oriented +/- 45 deg, one from another.

Chen and You, Proc. IMechE Vol. 220 Part G: J. Aerospace Engineering.

Important to remember!

- CGK formula is **simple** and **useful** but **does not consider any geometrical singularities** or **specificities!**

- **For in plane motion** => consider **3** dimensions instead of **6** in the formula to obtain a correct result
- ***Degree of hyperstatism*** *can be directly linked to the degree of mobility*